knowledge summary

RANDOM EXPERIMENT

An experiment is **random** if the following is true:

1. Its outcome depends on **chance**, meaning that the outcome of the experiment cannot be predicted with absolute certainty.
2. The set of all possible outcomes, called the **sample space**, can be listed before the experiment; this set is denoted by "Ω" otherwise known as "omega."

E.g. When a six-sided die whose faces are numbered from 1 to 6 is rolled, the sample space is $\Omega = \{1, 2, 3, 4, 5, 6\}$.

EVENT

An **event** is a **subset** of the sample space. An event is considered **simple** if it consists of only **one outcome** from the sample space.

E.g. 1) When drawing from a deck of 52 cards, "choosing a queen" is an event that corresponds to {queen of hearts, queen of spades, queen of diamonds, queen of clubs}.

2) When a coin is tossed, "landing on tails" is a simple event since it represents only one outcome {tails} of the sample space.

PROBABILITY OF AN EVENT

The **probability of an event** composed of several simple events is equal to the **sum of the probabilities** of each simple event.

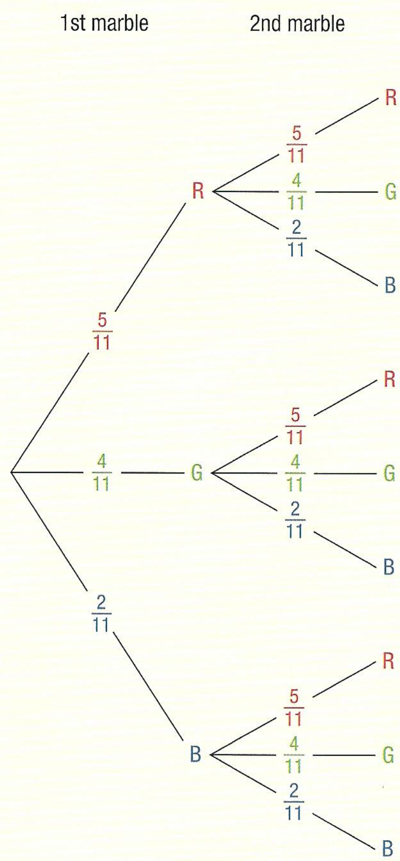
E.g. A drawer contains 8 knives, 10 forks and 12 spoons. Since "taking out a knife at random" and "taking out a fork at random" are two simple events, the probability of the event "taking out a knife or fork at random" is written as follows:

$$P(\text{knife or fork}) = P(\text{knife}) + P(\text{fork}) = \frac{8}{30} + \frac{10}{30} = \frac{18}{30} = \frac{3}{5}$$

RANDOM EXPERIMENT WITH SEVERAL STEPS

By including a probability on each branch of a tree diagram, you get a **probability tree**. In a random experiment with several steps, the probability of a simple event is equal to the **product of the probabilities** of all the intermediate steps that define this event.

E.g. A marble is drawn from a bag containing 5 red marbles, 4 green marbles and 2 blue marbles. The marble is then placed back in the bag, and another is selected.



Outcome	Probability
(R, R)	$\frac{5}{11} \times \frac{5}{11} = \frac{25}{121}$
(R, G)	$\frac{5}{11} \times \frac{4}{11} = \frac{20}{121}$
(R, B)	$\frac{5}{11} \times \frac{2}{11} = \frac{10}{121}$
(G, R)	$\frac{4}{11} \times \frac{5}{11} = \frac{20}{121}$
(G, G)	$\frac{4}{11} \times \frac{4}{11} = \frac{16}{121}$
(G, B)	$\frac{4}{11} \times \frac{2}{11} = \frac{8}{121}$
(B, R)	$\frac{2}{11} \times \frac{5}{11} = \frac{10}{121}$
(B, G)	$\frac{2}{11} \times \frac{4}{11} = \frac{8}{121}$
(B, B)	$\frac{2}{11} \times \frac{2}{11} = \frac{4}{121}$

WEIGHTED MEAN

The mean of a set of values that have varying levels of importance is called **weighted mean**.

E.g. A geography exam contains three parts. When a student's grade on each part and the relative importance of each part are taken into account, you get:

$$\text{Global result} = 0.75 \times 0.2 + 0.72 \times 0.3 + 0.88 \times 0.5$$
$$= 80.6\%$$

Geography exam

Part	Grade (%)	Weight (%)
A	75	20
B	72	30
C	88	50

knowledge in action

1 A box contains 26 tokens on which each of the 26 letters of the alphabet have been written. Three tokens are successively chosen at random and put back into the box. Calculate the probability of drawing:

a) three vowels

b) three consonants

c) a consonant followed by a vowel followed by a consonant

d) the letter F followed by a vowel followed by another vowel

e) the letter A followed by the letter B followed by the letter C

2 During a water quality test conducted on a river, a technician analyzes three water samples in succession to determine if they contain potable water or contaminated water. The probability of a sample being contaminated is 34%. Following are two events related to this situation:

A: only one sample is contaminated.

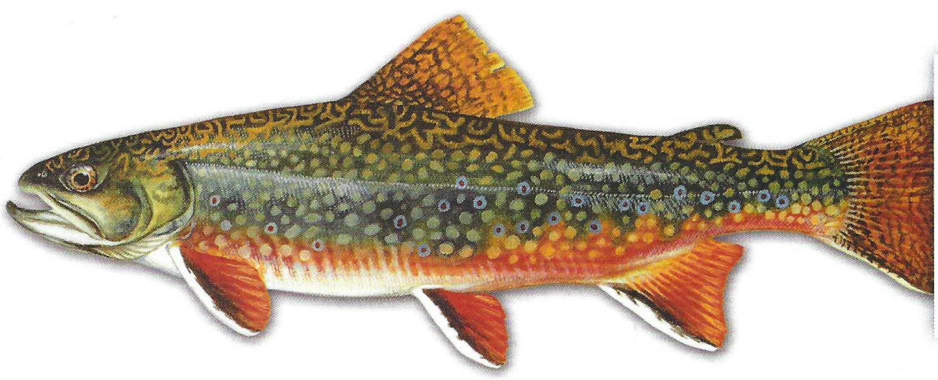
B: at least two samples are contamined.

a) Construct the probability tree associated with this situation.

b) Enumerate the results comprising event **A**.

c) Calculate the probability of event **B**.

d) Describe a simple event associated with this situation.



The brook trout requires a very clean aquatic environment in which to live. It needs clear, fresh and well-oxygenated waters. This species of fish is extremely sensitive to pollution. It is therefore an excellent indicator of the quality of its ecosystem.

3 A jar contains 5 red marbles, 3 yellow marbles and 2 black marbles. Three marbles are drawn consecutively. Calculate the probability of selecting the following:

a) one marble of each colour if the marbles are replaced after each selection

b) two red marbles and one black marble if they are not replaced

c) three marbles of the same colour if they are not replaced

d) two black marbles and one yellow marbles if they are replaced

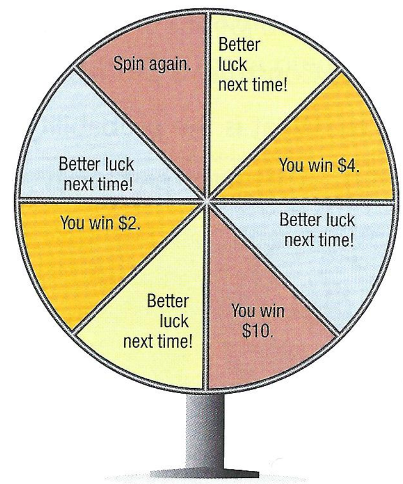
4 Maëva pays $3 and spins the wheel shown in the adjacent diagram. Determine the probability that she:

a) loses all her money on the first spin

b) wins $4 on the first spin

c) wins $10 on the second spin

d) spins the wheel 4 times



5 Naomi's French grades over the course of the first term are as follows:

Essay: 75%
Reading comprehension: 82%
Oral presentation: 79%

Determine Naomi's grade for the term given that the following is true:

- The essay portion is worth 40% of the term's grade.
- The reading comprehension portion is worth 45% of the term's grade.
- The oral presentation portion is worth 15% of the term's grade.

6 In a country, passenger vehicle licence plates are composed of three digits followed by three letters. Digits and letters may be repeated. If a licence plate is chosen at random, what is the probability that:

a) the first number is 7?

b) the two first letters are ZZ?

c) the three letters spell the word "FIN"?

d) the three digits make up the number 123?

The motto *Je me souviens* dates back to the 1880s. The Deputy Minister of Québec's Department of Crown Lands, Eugène-Étienne Taché, decided to inscribe it above the main entrance to the Parliament Building, which he designed. It became Québec's official motto in 1939.

7 A coin is tossed 4 times consecutively, and the side facing upward is noted each time.

a) Construct a probability tree that represents this situation.

b) What is the probability of obtaining 4 identical outcomes?

c) What is the probability of obtaining tails 2 times and heads 2 times?

The first coin ever made in Canada, in 1908, was the 50-cent coin, which bore the effigy of Edward VII, King of the United Kingdom and the countries of the Commonwealth in the early 1900s.

8 A bag contains 4 quarters and 5 loonies. A person randomly draws 2 coins consecutively from this bag. For each case, determine the probability of the event if the person:

1) does not put the first coin back in the bag
2) puts the first coin back in the bag

a) *P*(selecting coins with a total value of \$0.50)

b) *P*(selecting coins with a total value of \$1.25)

c) *P*(selecting coins with a total value of \$2.00)

9 Anti-spam software is designed to identify unwanted emails. Amie installs one of these programs on her computer. She estimates:

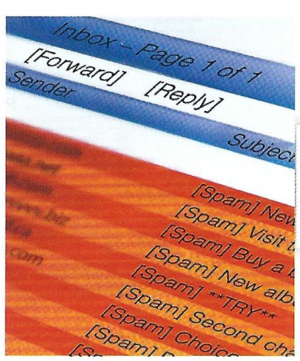
- a probability of 15% that the program will identify an email as unwanted when it is not
- a probability of 88% that the program will identify an email as unwanted when it is
- a probability of 12% that the program will identify an email as acceptable when it is not
- a probability of 85% that the program will identify an email as acceptable when it is

Amie receives an unwanted email followed by two acceptable emails. What is the probability that the program:

a) handled all emails correctly?

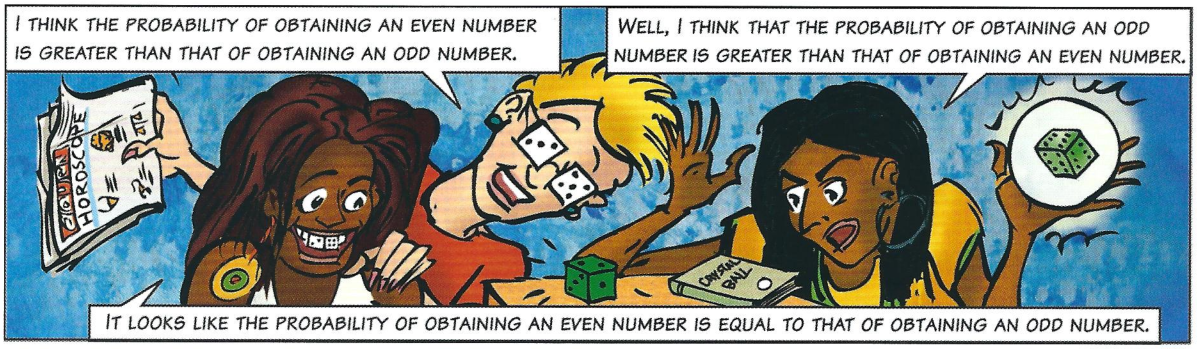
b) handled all emails incorrectly?

c) handled at least one email incorrectly?



The term "spam" generally refers to unwanted email. The first spam message was sent in 1978 to approximately 600 users of the ARPANET network, a predecessor to the Internet.

10 A 6-sided die whose faces are numbered 1 to 6 is rolled 5 times, and the following results are obtained: 2, 2, 4, 6 and 2. Following this experiment, three people make predictions regarding the result of the next roll.



Which of these three people is right? Explain your answer.

11 The gene associated with eye colour is made up of two alleles. When a child is conceived, each parent passes on one allele. The allele that leads to brown eyes is represented by the letter B and the allele that leads to blue eyes by the letter b. Chance determines which allele is passed on by each parent.

A man and a woman, each having both B and b alleles, have a child. There is:

- a probability of 45% that the father will pass on his B allele
- a probability of 55% that the father will pass on his b allele
- a probability of 40% that the mother will pass on her B allele
- a probability of 60% that the mother will pass on her b allele

a) What is the probability that this couple's child will receive:

1) two B alleles?

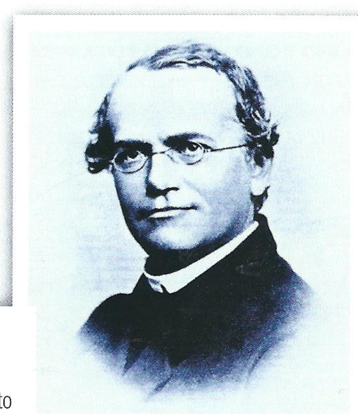
2) two b alleles?

3) a B allele and a b allele?

b) Considering that the presence of two b alleles is needed for the child to have blue eyes and that the presence of a single B allele suffices for the child to have brown eyes, calculate the probability that the couple's child will have eyes that are:

1) blue

2) brown



The mechanisms that govern the hereditary transmission of certain genes were discovered by the Austrian botanist and priest Gregor Mendel, following his experiments on pea plants. The laws relating to inheritance were named *Mendel's Laws* in his honour.

knowledge 4.1

LOGICAL CONNECTORS

The logical connectors "and" and "or" can be used to describe an event.

E.g. A 6-sided die whose faces are numbered from 1 to 6 is rolled.

- The outcome that corresponds to the event "obtaining a number that is even **and** prime" is 2 since it satisfies both characteristics stated simultaneously.
- The outcomes that correspond to the event "obtaining a number that is even **or** prime" are 2, 3, 4, 5 and 6 since each one satisfies one, the other or both of the characteristics stated.

VENN DIAGRAMS

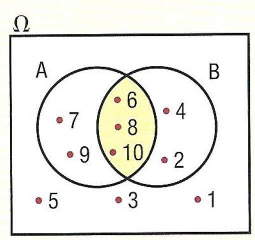
A Venn diagram allows you to graphically represent relationships among sets. In probability, each set generally corresponds to the outcomes that satisfy a given event.

In a Venn diagram, the following can be noted:

E.g. A 10-sided die whose faces are numbered from 1 to 10 is rolled. Below are two possible events:

Event A: obtaining a number greater than 5
Event B: obtaining an even number

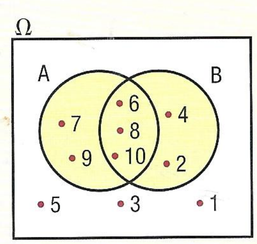
- The intersection of two sets A and B, written A ∩ B, is composed of elements that are common to both sets.

1) 

A ∩ B = {6, 8, 10}, which corresponds to numbers that are both greater than 5 **and** even.

The symbol ∩ is often associated with the logical connector "and."

- The union of two sets A and B, written A ∪ B, is composed of all the elements in both sets.

2) 

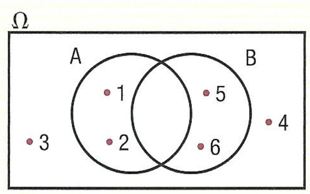
A ∪ B = {2, 4, 6, 7, 8, 9, 10}, which corresponds to numbers that are greater than 5 **or** even.

The symbol ∪ is often associated with the logical connector "or."

MUTUALLY EXCLUSIVE EVENTS AND NON-MUTUALLY EXCLUSIVE EVENTS

Two events are mutually exclusive if they cannot occur at the same time, meaning if $A \cap B = \varnothing$.

E.g. A 6-sided die whose faces are numbered from 1 to 6 is rolled. Event A, "obtaining a number less than 3," and event B, "obtaining a number greater than 4," are mutually exclusive since $A \cap B = \varnothing$.

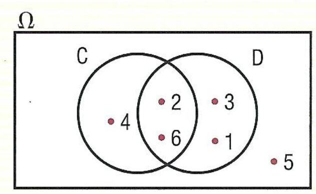


The probability of the event "obtaining a number less than 3 or a number greater than 4" is noted as follows:

$$P(A \cup B) = P(A) + P(B)$$
$$= \frac{2}{6} + \frac{2}{6}$$
$$= \frac{2}{3}$$

Two events are non-mutually exclusive if they can occur at the same time, meaning if $A \cap B \neq \varnothing$.

E.g. A 6-sided die whose faces are numbered from 1 to 6 is rolled. Event C, "obtaining an even number," and event D, "obtaining a factor of 6," are non-mutually exclusive since $C \cap D \neq \varnothing$.



The probability of the event "obtaining an even number or a factor of 6" is noted as follows:

$$P(C \cup D) = P(C) + P(D) - P(C \cap D)$$
$$= \frac{3}{6} + \frac{4}{6} - \frac{2}{6}$$
$$= \frac{5}{6}$$

You must subtract the probability of the intersection so that it is not counted twice.

Two mutually exclusive events whose union forms the set of all possible outcomes are complementary. The event that is complementary to event A is written A′ and the following equation is obtained:

$$P(A) + P(A') = 1$$

INDEPENDENT EVENTS AND DEPENDENT EVENTS

Two events A and B are independent if the occurrence of one does not influence the probability of the occurrence of the other.

E.g. A 6-sided die numbered from 1 to 6 is rolled twice. The probability that event A, "obtaining 4 on the first roll" and event B, "obtaining 3 on the second roll" will occur is noted as follows:

$$P(A \cap B) = \frac{1}{6} \times \frac{1}{6}$$
$$= \frac{1}{36}$$

Two events A and B are dependent if the occurrence of one influences the probability of the occurrence of the other.

E.g. Two marbles are drawn, without replacement, from a jar containing 49 marbles numbered 1 to 49. The probability that event C, "obtaining marble 7 on the first draw" and event D, "obtaining marble 5 on the second draw" will occur is noted as follows:

$$P(C \cap D) = \frac{1}{49} \times \frac{1}{48}$$
$$= \frac{1}{2352}$$

Since the 1st marble chosen is not put back in the vase

practice 4.1

1 For each case, determine whether events A and B are mutually exclusive. Explain your answer.

a) $P(A \cup B) = 0.75$, $P(A) = 0.45$, and $P(B) = 0.3$.

b) $P(A \cap B) = 0.1$

c) Events A and B are complementary.

d) $A \cap B = \varnothing$

2 A 6-sided die with faces numbered from 1 to 6 is rolled, and the side facing up is noted. The following are 3 possible events:

A: obtaining an even number

B: obtaining a 3

C: obtaining a 1 or a 6

a) Can it be said that:

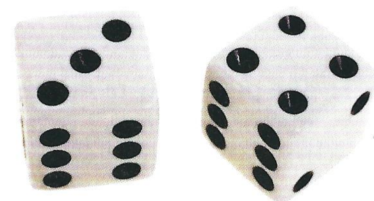
1) events A and B are mutually exclusive? Explain your answer.

2) events A and C are mutually exclusive? Explain your answer.

b) Calculate:

1) $P(A \cup B)$

2) $P(A \cup C)$



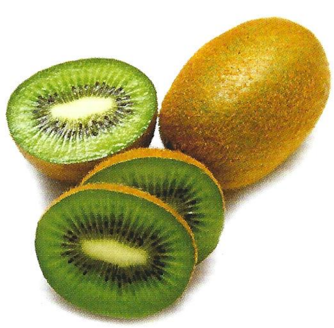
Although the origin of dice is not known for certain, it is known that their first appearance dates back to prehistory.

3 Random experiments with several steps are carried out. For each case, determine whether events A and B are dependent or independent.

a) A: drawing a green marble from a bag of marbles
B: landing on heads when tossing a coin

b) A: taking a kiwi at random from a bowl of fruit and eating it
B: taking a second kiwi at random from the same bowl

c) A: obtaining a 4 when rolling a die
B: obtaining a 4 when rolling the same die a second time

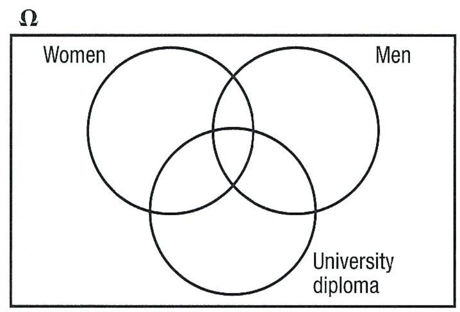


The kiwi originated in China but is mainly grown in New Zealand, Italy and France. Its name stems from the resemblance between its skin and that of the bird of the same name, which is the symbol of New Zealand.

4 In a company, 45 of the 95 employees are women. Among the employees, 24 women and 32 men have a university diploma.

a) Complete a Venn diagram similar to the one shown in the adjacent illustration to represent this situation.

b) If a person is chosen at random from these employees, calculate the probability that this person is a woman or has a university diploma.



5 A person walking on the street is chosen at random, and some of that person's characteristics are recorded. For each case, determine whether events A and B are mutually exclusive or non-mutually exclusive.

a) A: the person is a man.
B: the person is a woman.

b) A: the person has blue eyes.
B: the person has brown hair.

c) A: the person's height is greater than 1.5 m.
B: the person's height is greater than 1.7 m.

d) A: the person was born in June.
B: the person was born in the summer.

Eye colour comes from melanin, which also determines hair and skin colour. The less melanin there is in the iris, the lighter the eye colour will be and vice versa.

6 Determine whether the following equalities are true or false. If the equality is false, rewrite the right side to make it true.

a) $(A \cup B) \cup C = A \cup (B \cup C)$

b) $(A \cap B) \cap C = A \cap (B \cap C)$

c) $(A \cap B) \cup C = A \cap (B \cup C)$

d) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

e) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

f) $(A \cup B)' = A' \cup B'$

g) $A \cap \varnothing = A$

h) $A \cup \varnothing = \varnothing$

7 For each case, shade the area associated with the expression provided using a Venn diagram similar to the one shown in the adjacent illustration.

a) $A \cap B$

b) $A \cup C$

c) $A \cap B \cap C$

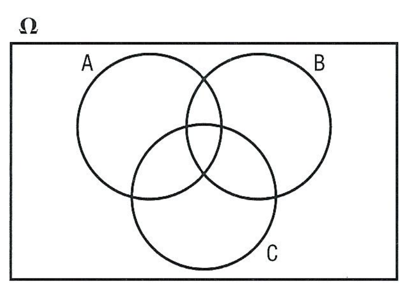
d) $(A \cap B) \cup C$

e) $A \cap (B \cup C)$

f) $A \cup (B \cap C)$

g) $A' \cap B'$

h) $A' \cap A$



8 Using a Venn diagram, show that:

a) $(A \cup B)' = A' \cap B'$

b) $(A \cap B)' = A' \cup B'$

9 In a group of 35 people:

- 7 people are left-handed.
- 15 people wear glasses.
- 17 people have brown hair.
- 3 people with brown hair wear glasses and are left-handed.
- 25 people have brown hair or wear glasses.
- 7 people have brown hair and wear glasses.
- 20 people are left-handed or have brown hair.
- 5 people wear glasses and are left-handed.

A person is chosen at random from this group. The following are 3 possible events:

A: choosing a left-handed person
B: choosing a person who wears glasses
C: choosing a person with brown hair

a) Represent this situation using a Venn diagram.

b) Express each of the following statements using set-builder notation.

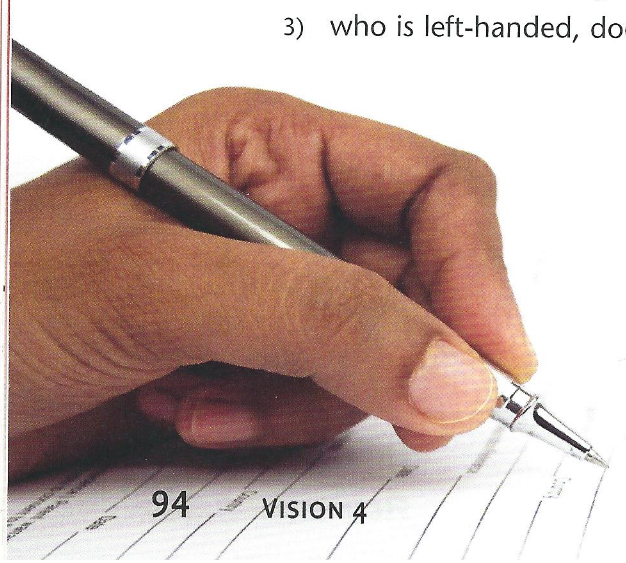
1) Choosing a person who wears glasses and is left-handed.
2) Choosing a left-handed person who wears glasses or has brown hair.
3) Choosing a person who has brown hair and wears glasses or a left-handed person who has brown hair.

c) Calculate:

1) $P(A \cup B)$
2) $P(A \cap B)$
3) $P(A \cup B \cup C)$
4) $P((A \cap B) \cap C)$
5) $P((A \cup B) \cap C)$
6) $P((B \cap C) \cup A)$

d) Calculate the probability of choosing a person:

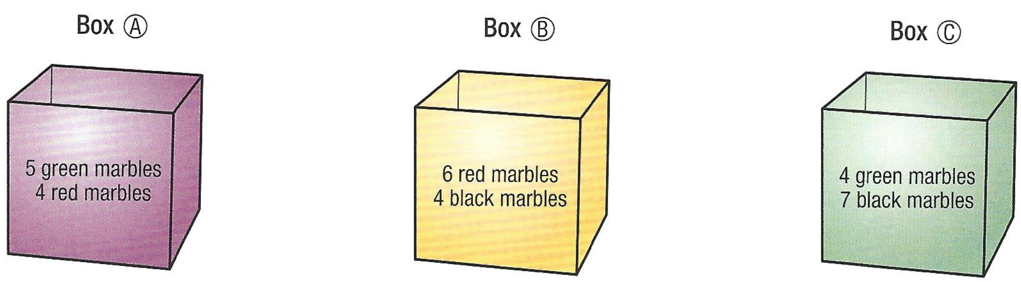
1) who wears glasses and does not have brown hair
2) who does not wear glasses and is not left-handed
3) who is left-handed, does not wear glasses and does not have brown hair



It is estimated that between 10% and 12% of the population is left-handed. Having a left-handed parent increases the probability that a child will also be left-handed.

10 If $A \cup B = A$, what does $A \cap B$ correspond to? Explain your answer.

11 Below is the content of three boxes:



A marble is drawn from Box Ⓐ and then replaced. If the marble selected is:

- red, a second selection is made from Box Ⓑ.
- green, a second selection is made from Box Ⓒ.

a) Are the compound events resulting from this experiment composed of dependent or independent events? Explain your answer.

b) What is the probability of the event:

1) "choosing a green marble on the first section followed by a red marble" on the second selection?
2) "choosing two marbles of the same colour"?
3) "choosing a single black marble"?

c) In this situation, describe two events that are:

1) mutually exclusive
2) non-mutually exclusive

12 Using a Venn diagram, represent three non-empty sets A, B and C, such that the following is true:

a) $A \cap B \neq \varnothing$, $B \cap C \neq \varnothing$, and $A \cap C = \varnothing$.

b) $A \cup B \cup C = A \cup B$ and $B \cap C = C$.

c) $A \cup B \cup C = C$ and $A \cap B = B$.

13 A 6-sided die with faces numbered 1 to 6 is rolled twice. The following are 2 possible events:

A: obtaining a sum that is even
B: obtaining a sum that is greater than or equal to 7

a) Are these two events mutually exclusive? Explain your answer.

b) Calculate:

1) $P(A)$ 2) $P(B)$ 3) $P(A \cap B)$ 4) $P(A \cup B)$

14 **BLOOD TYPES** The following is information regarding the compatibility of blood types during blood transfusions and on the distribution of blood types within the Canadian population.

Compatibility of blood types

Recipient \ Donor	O	A	B	AB
O	✔			
A	✔	✔		
B	✔		✔	
AB	✔	✔	✔	✔

Distribution of blood types within the Canadian population

O	A	B	AB
46%	42%	9%	3%

a) In Canada, what is the probability that a randomly selected person will be compatible with:

1) an O donor?
2) a B recipient?
3) an AB recipient?
4) an A donor?
5) an A recipient and a B recipient?
6) an AB donor or a B donor?

b) If two people are randomly selected, what is the probability that their blood types will be compatible in one way or the other?

Before the discovery of different blood types, many transfusions ended in failure because the recipient's immune system would consider the donor's blood cells, which were not compatible, as harmful organisms and destroy them.

15 Below are events related to the weather forecast for three consecutive days:

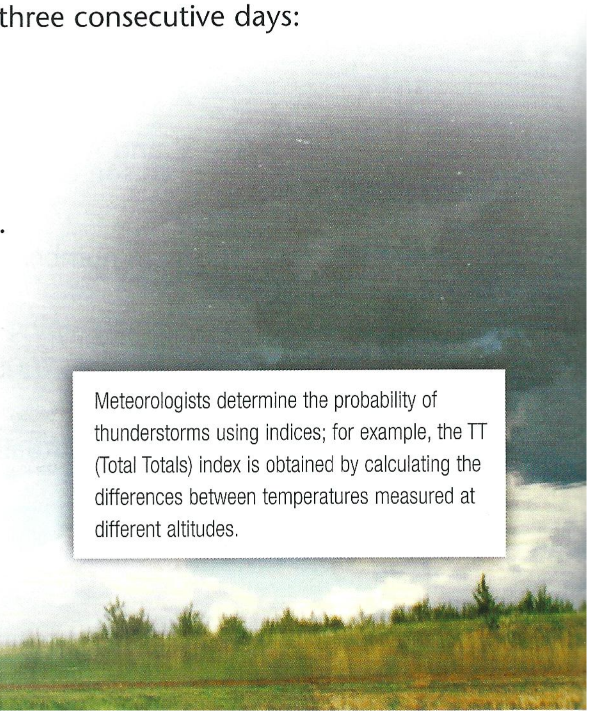
A: it will rain on Monday.
B: it will rain on Monday and Tuesday.
C: it will rain on Tuesday or Wednesday.

In addition: $P(\text{A}) = 0.45$, $P(\text{B}) = 0.3$ and $P(\text{C}) = 0.8$.

a) Calculate the probability that it will rain on:

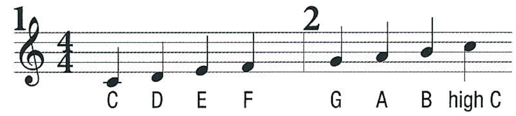
1) Tuesday
2) Wednesday
3) Monday or Wednesday
4) Monday, Tuesday and Wednesday

b) Calculate the probability that it will rain on at least one of the three days.



Meteorologists determine the probability of thunderstorms using indices; for example, the TT (Total Totals) index is obtained by calculating the differences between temperatures measured at different altitudes.

16 A student chooses three notes randomly and without repetition from the eight notes of the C major scale shown in the adjacent illustration.



The chord composed of the three notes is defined as follows:

- It is a C major if it is composed only of notes from the set {C, E, G, high C}.
- It is a G major if it is composed of the notes E, G and B.

The following are 3 events related to this situation:

A: obtaining a C major chord
B: obtaining a chord that contains the note G
C: obtaining a G major chord

a) Translate the following events into words.

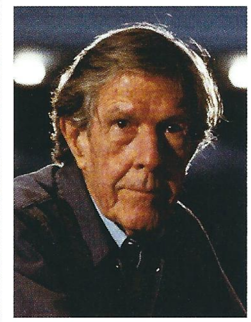
1) $A \cup B$ 2) $A \cap C$ 3) $B \cap C$

b) Using events A, B and C, identify:

1) two pairs of non-mutually exclusive events
2) one pair of mutually exclusive events

c) Calculate the probability of obtaining the following chord:

1) C major
2) C major that contains the note G
3) G major or a chord that contains the note G



Some composers have introduced chance into their works. For instance, John Cage (1912-1992) composed some of his pieces using random generation.

17 The adjacent table presents the distribution of employees at a company based on sex and mother tongue.

A member of this company's staff is randomly selected for training. The following are three possible events:

A: the staff member is a man.
B: the staff member is a woman.
C: the staff member is Anglophone.

A company's employees

Mother tongue \ Sex	Man	Woman	**Total**
French	34	77	**111**
English	21	11	**32**
Other	10	4	**14**
Total	**65**	**92**	**157**

a) Illustrate the preceding events in a Venn diagram, and include the appropriate numbers.

b) Calculate:

1) $P(A \cup B)$ 2) $P(A \cup C)$ 3) $P(B \cup C)$ 4) $P(A \cap C)$ 5) $P(B \cap C)$

A team is randomly formed of three employees. The following are 3 possible events:

D: choosing an Anglophone staff member with the 1st selection
E: choosing a woman with the 2nd selection
F: choosing a Francophone staff member with the 3rd selection

c) Are events D, E and F dependent or independent? Explain your answer.

d) Calculate $P(D \cap E \cap F)$.