5.3 Events

ACTIVITY 1 Universal set – Events

A fair die is rolled once.

a) Describe the universal set Ω, in other words the set of all possible outcomes.

____

b) Describe the event
1. A: «rolling an even number». ______________________
2. B: «rolling a number greater than or equal to 1» ______________________
3. C: «rolling a number greater than 6» ______________________

c) Which of the three events defined in b) is
1. certain. ______________ 2. impossible. ______________

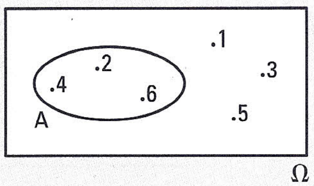
UNIVERSAL SET – EVENT

- An experiment is **random** when the outcome of this experiment cannot be predicted with certainty.

  Ex.: The experiment consisting in rolling a die and observing the outcome is a random experiment.
- The **universal set** associated with a random experiment is the set of all possible outcomes of the experiment.
  This set is written Ω (read «omega»).

  Ex.: The possible outcomes of the experiment above are 1, 2, 3, 4, 5 or 6.
  Therefore, we have $\Omega = \{1, 2, 3, 4, 5, 6\}$.
- An **event** associated with a random experiment is a subset of the set Ω of all possible outcomes.

  Ex.: A die is rolled once.
  The event A: «rolling an even number» is described extensively by $A = \{2, 4, 6\}$ and on the right using a **Venn diagram**.



- Among events, we distinguish
  – the **certain** event, written Ω, event which always occurs.
  – the **impossible** event, written $\varnothing$, event which never occurs.

1. For each of the following random experiments, determine the universal set Ω. (Use a tree diagram if necessary).

a) A coin is tossed twice.

____

b) A die is rolled twice.

____

____

c) A coin is tossed three times.

____

2. A die is rolled twice.

a) Describe extensively the following events.

1. A: «rolling a sum equal to 10». ______
2. B: «rolling a product equal to 12». ______
3. C: «getting an even number on the first roll and a sum equal to 7». ______

b) Describe in words the following events.

1. A = {(6,1), (6,2), (6,3) (6,4), (6,5), (6,6)} ______
2. B = {(1,4), (2,3), (3,2) (4,1)} ______
3. C = {(1,1), (1,2), (1,3), (2,1), (2,2), (3,1)} ______

3. Two people chosen at random on a street corner are asked about their opinion regarding the merger of two cities. Each person can declare that they agree (a), they disagree (d) or refuse to answer (r).

a) Describe extensively the universal set Ω.

______

b) Describe extensively the following events.

A: «the two people agree». ______

B: «the 1st person agrees». ______

C: «the two people give the same answer». ______

D: «at least one of the two people agrees». ______

c) Describe in words the following events.

A = {(*r*, *r*)} ______

B = {(*a*, *d*), (*d*, *d*), (*r*, *d*)} ______

C = {(*a*, *d*)} ______

D = {(*d*, *d*), (*d*, *r*), (*r*, *d*), (*r*, *r*)} ______

4. At basketball, Valerie shoots the ball at the basket three times consecutively. Let S represent a success and F a failure.

a) Describe extensively the universal set Ω of all possible outcomes.

______

b) Describe extensively the following events:

A: «Valerie succeeds on her 1st shot and fails on the other two.». ______

B: «Valerie succeeds on each shot.». ______

C: «Valerie succeeds on the first shot.». ______

c) Describe in words the following events.

A = {(F, F, S)} ______

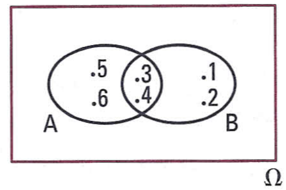
B = {(F, F, F)} ______

C = {(S, S, F), (S, F, F), (F, S, F), (F, F, F)} ______

5.4 Operations between events

ACTIVITY 1 The event A and B

A die is rolled once. Consider the following events.
A: "rolling a number greater than 2".
B: "rolling a number less than 5".



a) Describe extensively the following events.

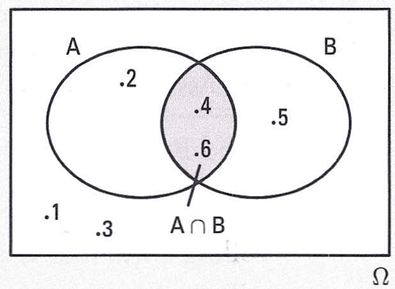
1. A = ____________________ 2. B = ____________________

b) What is the set of possible outcomes if we know that both events A and B occurred? ________

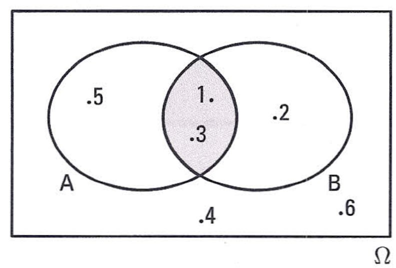
INTERSECTION

Given a random experiment and two events A and B, the event A and B, written **A ∩ B** (read: A intersect B), is the event that occurs if, and only if, A occurs and B occurs.

Ex.: A die is rolled once. Consider the following events.
A: "rolling an even number".
B: "rolling a number greater than 3".
The event A ∩ B is defined by: "rolling an even number greater than 3".
Note that:
A = {2, 4, 6}, B = {4, 5, 6} and A ∩ B = {4, 6}.



1. A die is rolled once. Let A and B be the events defined by:
A: "rolling an odd number"; B: "rolling a number less than 4".



a) Represent the events Ω, A and B on a Venn diagram.

b) 1. Color the region associated with the event A ∩ B on this diagram.
2. Describe extensively the event A ∩ B. ____________
3. Describe in words the event A ∩ B. ____________________

2. A coin is tossed twice.

a) Describe extensively the following events.
1. A: "getting tails on the 1st toss". ____________________
2. B: "getting tails on the 2nd toss". ____________________

b) Describe in words the event A ∩ B. ____________________

c) Describe extensively the event A ∩ B. ____________________

3. A card is drawn from a 52-card deck. Let A and B be two events defined by:
A: "drawing a queen"; B: "drawing a heart".
Describe in words the event $A \cap B$. ______________________________

ACTIVITY 2 Incompatible events

A die is rolled once. Let A and B be the events defined by:
A: "rolling a number less than 3"; B: "rolling a number greater than 4".

a) Describe extensively the following events.
1. A = ________________ 2. B = ________________

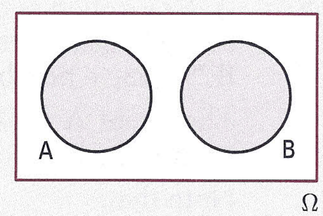
b) Represent the events Ω, A and B on a Venn diagram.

c) 1. Describe in words the event $A \cap B$. ______________________________
2. What can you say about the event $A \cap B$? ______________________________

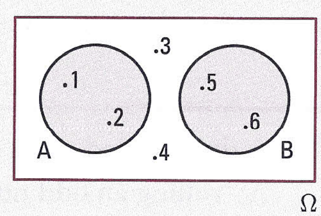
INCOMPATIBLE EVENTS

Two events A and B are **incompatible** (or **mutually exclusive**) if it is impossible for these two events to occur simultaneously. $A \cap B$ is then the impossible event.

Thus, A and B are incompatible $\Leftrightarrow A \cap B = \varnothing$



Ex.: In the experiment where a die is rolled once, the events
A: "rolling a number less than or equal to 2"
and B: "rolling a number greater than or equal to 5" are incompatible since $A \cap B = \varnothing$.



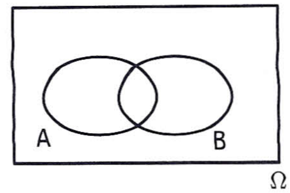
4. A fair die is rolled once. Let A, B, C and D be the events defined by;
A: "rolling an even sum"; B: "rolling a sum equal to 5";
C: "rolling a sum greater than 10"; D: "getting 6 on the 1st roll".
True or false?

a) A and B are incompatible. __________ **b)** A and C are incompatible. __________
c) A and D are incompatible. __________ **d)** B and C are incompatible. __________
e) B and D are incompatible. __________ **f)** C and D are incompatible. __________

ACTIVITY 3 Event A or B

A die is rolled once. Consider the following events.

A: "rolling a number less than 5" and B: "rolling an odd number".



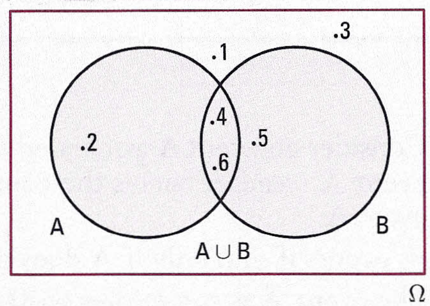
a) Describe extensively the following events.

1. A = ______________________ 2. B = ______________

b) What is the set of all possible outcomes if we know that event A occurred or that event B occurred? ______________________

UNION

Given a random experiment and two events A and B, the event A or B, written $A \cup B$ (read: A union B), is the event that occurs if, and only if, A occurs **or** B occurs.



Ex.: A die is rolled once. Consider the following events.
A: "rolling an even number".
B: "rolling a number greater than 3".
The event $A \cup B$ is defined by: "rolling an even number or a number greater than 3".
Note that: $A = \{2, 4, 6\}$, $B = \{4, 5, 6\}$ and $A \cup B = \{2, 4, 5, 6\}$.

5. A die is rolled twice. Let A and B be the events defined by:
A: "getting 6 on the 1st roll"; B: "getting 6 on the 2nd roll".

Describe extensively the following events.

a) A ______________________

b) B ______________________

c) $A \cup B$ ______________________

d) $A \cap B$ ______________________

6. Nam Soo writes an English exam and a French exam. We observe, for each exam, his performance, namely if he passes of fails. The couple (F, S) indicates that he failed in English and passed in French.

a) Describe extensively the following events.
1. A: "Nam Soo passes the English exam". ______________________
2. B: "Nam Soo passes the French exam". ______________________

b) Describe in words the following events.
1. $A \cap B$ ______________________
2. $A \cup B$ ______________________

____

c) Describe extensively the following events.
1. $A \cap B$ ______________ 2. $A \cup B$ ______________

ACTIVITY 4 Event contrary to event A

The contrary event to an event A is designated by $\bar{A}$.
A die is rolled once.

a) Describe in words the contrary to the following events.

1. A_1: "rolling an even number". ______
2. A_2: "rolling a 3". ______
3. A_3: "rolling a number less than 3". ______

b) Describe extensively the contrary to the following events.

1. $A_1 = \{2, 4, 6\}$ ______ 2. $A_2 = \{3\}$ ______ 3. $A_3 = \{1, 2\}$ ______

CONTRARY EVENT

Consider an event A associated with a random experiment. The event $\bar{A}$ (read: A bar) is the **contrary event (complementary)** to event A.

$\bar{A}$ occurs if, and only if, A does not occur.

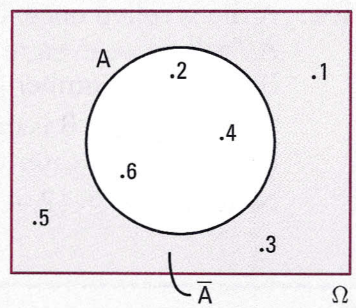
The event $\bar{A}$ is sometimes written A′ (read: A prime).

Ex.: A die is rolled once. Consider the event A: "rolling an even number".

The event $\bar{A}$ is defined by: "rolling an odd number".

Note that $A = \{2, 4, 6\}$ and $\bar{A} = \{1, 3, 5\}$.

We have: $A \cap \bar{A} = \varnothing$ and $A \cup \bar{A} = \Omega$.



7. A die is rolled once. Let A_1, A_2, A_3 be the events defined by:
A_1: "rolling an odd number"; A_2: "rolling a 5"; A_3: "rolling a number greater than 3".

a) Describe in words the following events.

1. $\bar{A}_1$ ______
2. $\bar{A}_2$ ______
3. $\bar{A}_3$ ______

b) Describe extensively the following events.

1. $\bar{A}_1$ ______ 2. $\bar{A}_2$ ______ 3. $\bar{A}_3$ ______

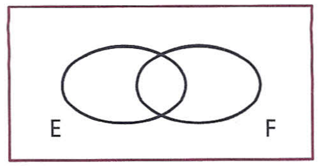
ACTIVITY 5 De Morgan's laws

Consider, in a group of tourists, the set E of tourists who speak English and the set F of those who speak French.

a) 1. Represent $\overline{E \cup F}$ on the diagram on the right.

2. Describe in words $\overline{E \cup F}$. ____________________

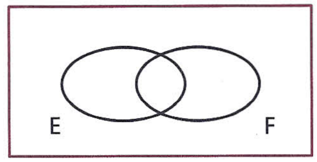
____



b) On the diagram on the right,

1. shade $\bar{E}$ using horizontal lines. 2. shade $\bar{F}$ using vertical lines.

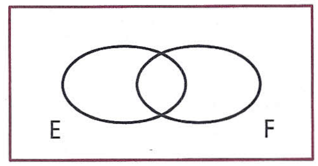
3. how is the set $\bar{E} \cap \bar{F}$ represented? ____________________



c) Compare $\overline{E \cup F}$ and $\bar{E} \cap \bar{F}$. ____________________

d) 1. Represent $\overline{E \cap F}$ on the diagram on the right.

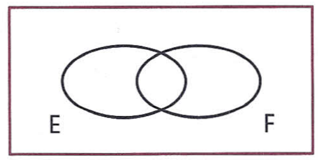
2. Describe $\overline{E \cap F}$ in words. ____________________



e) On the diagram on the right,

1. shade $\bar{E}$ using horizontal lines. 2. shade $\bar{F}$ using vertical lines.

f) Compare $\overline{E \cap F}$ and $\bar{E} \cup \bar{F}$. ____________________



DE MORGAN'S LAWS

Let A and B be two events associated with a random experiment. We have:

$\overline{A \cup B} = \bar{A} \cap \bar{B}$ and $\overline{A \cap B} = \bar{A} \cup \bar{B}$

Thus, the complement of the union is equal to the intersection of the complements and the complement of the intersection is equal to the union of the complements.

8. A die is rolled once. Consider the following events.
A: "rolling an even number" and B: "rolling a number greater than 3".

a) Describe extensively the following events.

1. $A \cup B$ ________ 2. $A \cap B$ ________ 3. $\overline{A \cup B}$ ________ 4. $\overline{A \cap B}$ ________

b) Describe extensively the following events.

1. $\bar{A}$ ________ 2. $\bar{B}$ ________ 3. $\bar{A} \cup \bar{B}$ ________ 4. $\bar{A} \cap \bar{B}$ ________

c) Verify De Morgan's two laws.

1. $\overline{A \cup B} = \bar{A} \cap \bar{B}$ 2. $\overline{A \cap B} = \bar{A} \cup \bar{B}$

d) Describe in words the following events.

1. $\bar{A} \cap \bar{B}$ ____________________
2. $\overline{A \cap B}$ ____________________
3. $\bar{A} \cup \bar{B}$ ____________________
4. $\overline{A \cup B}$ ____________________

9. A student is chosen at random from a group. Consider the following events.
F: "the student is a girl" and G: "the student wears glasses".

Describe in words the following events.

a) $F \cup G$ ______________________________

b) $F \cap G$ ______________________________

c) $\bar{F}$ ______________________________

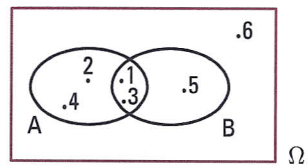
d) $F \cap \bar{G}$ ______________________________

e) $\bar{F} \cap \bar{G}$ ______________________________

f) $\bar{F} \cup \bar{G}$ ______________________________

ACTIVITY 6 The event A minus B

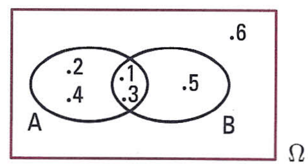
A die is rolled once. Consider the following events.
A: "rolling a number less than 5" and B: "rolling an odd number".



a) 1. Describe extensively the event $A \cap \bar{B}$. ______________________________

2. Describe in words the event $A \cap \bar{B}$.

____



b) 1. Describe extensively the event $\bar{A} \cap B$. ______________________________

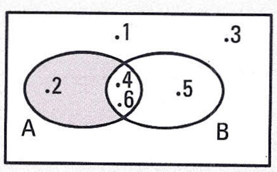
2. Describe in words the event $\bar{A} \cap B$.

____

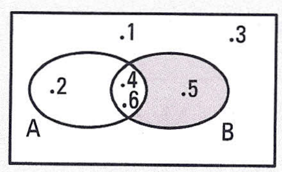
DIFFERENCE EVENT

Given a random experiment and two events A and B, the event "A minus B", written A\B (read: A minus B), is the event that occurs if, and only if, A occurs and B does not occur.

Ex.: A die is rolled once. Consider the following events.
A: "rolling an even number" and B: "rolling a number greater than 3".
– The event A\B is defined by: "rolling an even number less than or equal to 3".
– The event B\A is defined by: "rolling an odd number greater than 3".



$A \backslash B = \{2\}$



$B \backslash A = \{5\}$

We have: $A \backslash B = A \cap \bar{B}$ and $B \backslash A = \bar{A} \cap B$

10. A student is chosen at random from a group writing English and biology exams. Consider the following events.

A: "the student passes in English" and B: "the student passes in biology".

Describe in words the following events.

a) $A \cup B$ ______________________________

b) $A \cap B$ ______________________________

c) $\overline{A}$ ______________________________

d) $A \backslash B$ ______________________________

e) $B \backslash A$ ______________________________

f) $\overline{A} \cap \overline{B}$ ______________________________

g) $\overline{A} \cup \overline{B}$ ______________________________

11. A student from the class is chosen at random. Consider the following events.

B: "the student is a boy" and R: "the student is right-handed".

Write the following events symbolically.

a) The student is a right-handed boy. ______________________________

b) The student is a left-handed boy. ______________________________

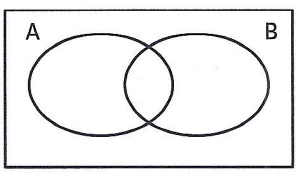
c) The student is a right-handed girl. ______________________________

d) The student is a left-handed girl. ______________________________

12. Let A and B be two events associated with a random experiment.

1. Represent the following events on a Venn diagram.
2. Describe each event using the operators ∩, ∪ or ¯.

a) A and B occur.



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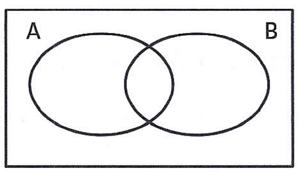
b) A or B occurs.



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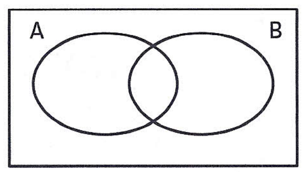
c) B does not occur.



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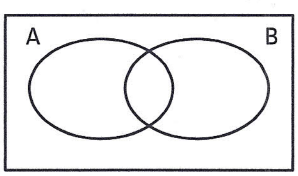
d) A occurs and B does not occur.



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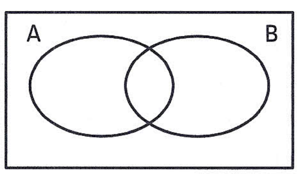
e) A does not occur and B occurs.



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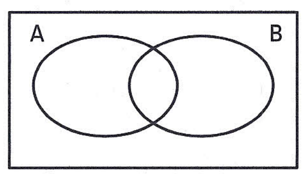
f) A and B do not occur.



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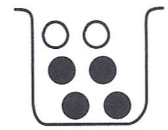
5.5 Probability of an event

ACTIVITY 1 Probability of an outcome



A jar contains 6 balls: 4 black ones and 2 white ones. A ball is drawn from the jar and its color is recorded. W represents the outcome "white" and B the outcome "black".

a) Describe the set Ω of all possible outcomes. ______________________

b) Are the outcomes equally likely? ______________________

c) Calculate 1. the probability of drawing a black ball, written P(B); ______________________

2. the probability of drawing a white ball, written P(W). ______________________

d) If the jar contains 6 balls: 3 black ones and 3 white ones, answer questions a), b) and c) again.

a) ______________ **b)** ______________ **c)** ______________

ACTIVITY 2 Probability of an event

A die is rolled once and the outcome is recorded.

a) Describe the set Ω of possible outcomes. ______________________

b) Are the possible outcomes equally likely? ______________________

c) Consider the event A: "rolling an even number".

1. Describe event A extensively. ______________________
2. Calculate the probability, written P(A), that event A occurs. ______________________

d) Consider the event B: "rolling a number less than 5".

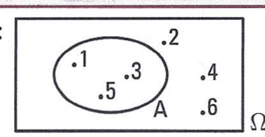
1. Describe event B extensively. ______________ 2. Calculate P(B). ______________

PROBABILITY OF AN EVENT IN AN EQUALLY LIKELY SITUATION

Let Ω be the universal set associated with a random experiment and A an event. If the possible outcomes of the experiment are equally likely, the probability of event A, written P(A), is the ratio between the number of outcomes favourable to the occurrence of A and the number of possible outcomes.

$$P(A) = \frac{\text{Number of outcomes favourable to event A}}{\text{Number of possible outcomes}} = \frac{n(A)}{n(\Omega)}$$

Ex.:



A die is rolled once.

A: "rolling an odd number".

$A = \{1, 3, 5\}$, $P(A) = \frac{3}{6} = \frac{1}{2}$

Note: $n(E)$ represents the number of elements in E, called **cardinality** of E.

1. A card is drawn from a 52-card deck. What is the probability of drawing

a) a king? ______________ **b)** a spade? ______________ **c)** the king of spade? ______________

2. A coin is tossed twice. What is the probability of getting a total of

a) one tails? ______________ **b)** two tails? ______________ **c)** at least one tails? ______________

3. In a family with three children, calculate the probability of the following event:

a) the oldest child is a boy. ______________ **b)** there is a total of 3 boys. ______________

c) there is a total of at least one boy. _______ **d)** there are more boys than girls. _________

4. A die is rolled twice. Calculate the probability of the following event:

a) we get a "6" on the first roll; ______________ **b)** we get a "6" on the second roll; ________

c) we get a "6" on the first or second roll; _____ **d)** we get a "6" on each roll; ______________

e) we get no "6". __________________________

5. A die is rolled twice. Calculate the probability of the following event:

a) the sum of the outcomes is equal to 10. ______________________________

b) the sum of the outcomes is greater than 10. ______________________________

c) the product of the outcome is equal to 12. ______________________________

d) we get the same outcome on each roll. ______________________________

e) the 1st outcome is even and the sum is equal to 7. ______________________________

6. A die is rolled and a coin is tossed at the same time. Calculate the probability of the following event:

a) the die shows "6". ______________________________

b) the coin shows "tails". ______________________________

c) the die shows "6" and the coin shows "tails". ______________________________

d) the die does not show "6" and the coin does not show "tails". ______________________________

e) the number on the die is even and the coin shows "tails". ______________________________

7. A coin is tossed three times. Calculate the probability of the following event:

a) we get tails on each toss; ________________ **b)** we get tails on the 3rd toss; ___________

c) we get a total of at least one tails; _________ **d)** we get tails on the 1st and 3rd tosses; ___

e) we get tails on the 1st or 3rd toss. _________

8. In a family with three children, calculate the probability of the following events.

a) The oldest child is a boy. ______ **b)** The oldest and the youngest children are boys. ___

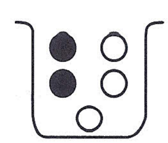
c) There is a total of two boys. ___ **d)** There is at least one boy. ______________

9. In a family with four children, calculate the probability of the following events.

a) The oldest child is a boy. ______ **b)** The oldest and the youngest children are boys. ___

c) There is a total of two boys. ___ **d)** There is at least one boy. ______________

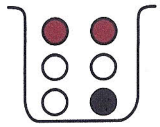
10. A jar contains 3 white balls and 2 black balls. Two balls are drawn successively from the jar and we get a white ball on the 1st draw. What is the probability that the 2nd ball drawn is white if the draw is



a) with replacement? ___________ **b)** without replacement? ___________

ACTIVITY 3 Chances for – Chances against

The jar on the right contains 2 orange balls, 3 white balls and one black ball. A ball is drawn from the jar. Consider the following events: R: "drawing an red ball", W: "drawing a white ball" and B: "drawing a black ball".



a) Calculate. 1. P(R) ______ 2. P(W) ______ 3. P(B) ______

b) The chances of the event "drawing an orange ball" occurring are 2 against 4 (written 2 : 4) i.e. 2 favourable outcomes against 4 unfavourable outcomes. What are the chances that the following events occur?

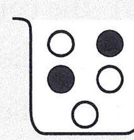
1. "drawing a white ball". ______
2. "drawing a black ball". ______

CHANCES FOR – CHANCES AGAINST

During a random experiment, consider an event A.

- The **chances for** an event occurring are defined by the ratio,
  number of outcomes favourable to A : number of outcomes unfavourable to A.
- The **chances against** an event occurring are defined by the ratio,
  number of outcomes unfavourable to A : number of outcomes favourable to A.

Ex.: A ball is drawn from the jar on the right, which contains 3 white balls and 2 black balls.



- The **chances for** drawing a white ball are 3 against 2 or 3:2.
- The **chances against** drawing a white ball are 2 against 3 or 2:3.

11. A die is rolled once.

a) What are the chances for rolling a 6? ______

b) What are the chances against rolling a 6? ______

12. A card is drawn from a 52-card deck. What are the chances

a) for drawing a face card? ______ **b)** for drawing a queen? ______

c) against drawing a face card? ______ **d)** against drawing a king? ______

13. The chances of an event A occurring are *a* against *b*. Determine

a) P(A); ______ **b)** $P(\bar{A})$. ______

14. A card is drawn from a 52-card deck. What are the chances of drawing

a) a queen? ______ **b)** a heart? ______ **c)** the queen of hearts? ______

15. What are the chances of getting a right answer for a student who chooses an answer at random for

a) a True or False type question? ______

b) a multiple choice question with five choices? ______

16. Chances that horse A wins a race are estimated at 3 : 2, i.e. "3 against 2".

a) Interpret this ratio. ______________________________

____

b) What is the probability that the horse wins the race? ______________________________

ACTIVITY 4 Theoretical probability, empirical probability and subjective probability

a) Consider a fair coin (not loaded). What is, theoretically, the probability that the coin shows tails when it is tossed once?

____

b) A coin is fake. Find a method for obtaining the probability of getting tails.
The probability obtained is called **empirical** probability.

____

____

____

c) The probability that the unemployment rate increases next month is unknown. Some economics experts can estimate this probability based on observation of certain indices. What is the probability obtained in this way called?

____

THEORETICAL PROBABILITY – EMPIRICAL PROBABILITY – SUBJECTIVE PROBABILITY

- A **theoretical** (or a priori) **probability** is a probability determined in advance, without performing any experiment.

  Ex.: The probability that a fair coin shows tails when it is tossed once is $\frac{1}{2}$.

- The **empirical** (or frequential) **probability** of an event E, written P(E), corresponds to the relative frequency of this event when the experiment is repeated a very large number of times.

$$P(E) = \frac{n(E)}{n}$$

  n designates the number of experiments and n(E) the number of times that event E occurred.

  Ex.: The probability that a fake coin shows tails is unknown. The coin is tossed 200 times and we observe a total of 112 tails. The empirical probability that the coin shows tails is $0.56\left(\frac{112}{200}\right)$.

- The **subjective probability** of an event is considered when it is impossible to establish the theoretical probability or the empirical probability.

  Ex.: An economist estimates there is a 40% chance that Canada will enter a recession next year. This subjective probability, based on economic indices, is established according to the expertise of the economist.

17. A meteorologist estimates at 20% the chances of having rain tomorrow.

a) Is the probability established by the meteorologist empirical or subjective? ______________

b) What is the probability that it will not rain tomorrow? ______________

18. A coin is fake. The table on the right gives the number of tails observed after n consecutives tosses.

Number n of tosses	Number of tails observed
10	6
25	13
50	23
100	48

a) Using the table on the right, we can approximate the probability that the coin shows tails when it is tossed. Approximate the probability that the coin shows tails after performing

1. 10 tosses; ______________ 2. 25 tosses; ______________
3. 50 tosses; ______________ 4. 100 tosses. ______________

b) Are the probabilities established in a) empirical or subjective? ______________

c) What is the best approximation of the probability that the coin shows tails? Justify your answer.

____

19. In each of the following cases, indicate the type of probability (theoretical, empirical, subjective).

a) The probability of rolling a sum equal to 7 when a die is rolled twice is equal to $\frac{1}{6}$.

____

b) The probability that a camper chosen at random at a summer camp is a boy is equal to $\frac{5}{12}$.

____

c) The probability that there is a total of 2 boys in a family with 3 children is equal to $\frac{3}{8}$.

____

d) The probability that on the 1st of January of next year the value of the Canadian dollar exceeds that of the US dollar is equal to $\frac{2}{3}$.

____

e) A survey among Canadian families shows that 10% of families have 3 or more children.

____

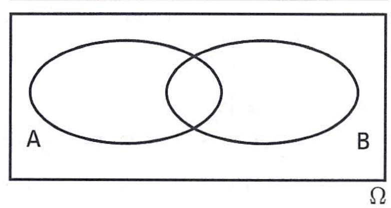
f) A researcher predicts that from 2020, one out of two Canadian will one day suffer from cancer.

____

ACTIVITY 5 Double entry table – Venn diagram

All 25 students of a class wrote English and biology exams. There is a total of 20 students who passed in English, 18 who passed in biology and 15 who passed both exams. A student is selected at random. Consider the events A: "the student passes in English" and B: "the student passes in biology".

	B	$\overline{B}$	Total
A	15		20
$\overline{A}$			
Total	18		25



a) Complete the double entry table on the right by writing the appropriate number of students in each box.

b) Complete the diagram on the right by writing the appropriate number of students in each region.

c) What is the probability, for a student selected at random, that the student

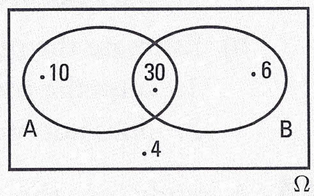
1. passes in English only? ______________
2. passes in biology only? ______________
3. fails in English? ______________
4. fails in biology? ______________
5. fails both exams? ______________
6. fails in English only? ______________

DOUBLE ENTRY TABLE –VENN DIAGRAM

- A double entry table is used to present the outcomes favourable and unfavourable to the occurrence of 2 events A and B.

	B	$\bar{B}$	Total
A	30	10	40
$\bar{A}$	6	4	10
Total	36	14	50

- A Venn diagram is used to visualize the outcomes favourable and unfavourable to the occurrence of 2 events A and B or of 3 events or more A, B, C, …



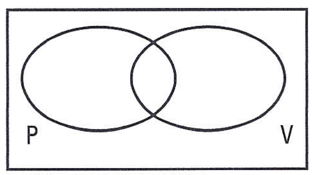
Ex.: From the double entry table or the Venn diagram on the right, we deduce that there is a 80% $\left(\frac{40}{50}\right)$ chance that A occurs, 72% $\left(\frac{36}{50}\right)$ chance that B occurs, $\left(\frac{30}{50}\right)$ chance that A and B occur, 20% $\left(\frac{10}{50}\right)$ chance that A occurs but B does no occur …

20. In a music school with 120 students, there are 90 students learning piano, 40 students learning violin and 30 students learning both these instruments.
A student is selected at random. Consider the events P: "the chosen student is learning piano" and V: "the chosen student is learning violin".

a) Represent the situation with

1. a double entry table.

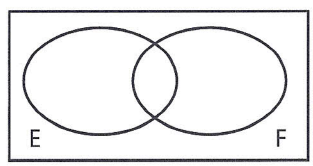
2. a Venn diagram.



b) What is the probability that the chosen student is learning

1. only piano? ______________________
2. neither of these two instruments? ______

21. A study shows that, in a group of 50 tourists, 28 speak English, 20 speak French and 10 speak English and French. A tourist is chosen at random from the group. Consider the following events:
E: "speaking English"; F: "speaking French".



a) Represent the situation on the Venn diagram on the right.

b) Calculate the probability of each of the following events.

1. The tourist speaks English. ______________________
2. The tourist speaks English but not French. ______________________
3. The tourist speaks English and French. ______________________
4. The tourist speaks English or French. ______________________
5. The tourist speaks neither English nor French. ______________________
6. The tourist does not speak English or does not speak French. ______________________

22. After a survey of 200 consumers, people who answered were divided according to whether or not they have seen a commercial and whether or not they have bought the product.

Commercial \ Product	Bought	Did not buy	Total
Saw	20	60	80
Did not see	30	90	120
Total	50	150	200

a) Estimate the probability that a consumer

1. buys the product. ______
2. sees the commercial. ______
3. sees the commercial and does not buy the product. ______
4. does not see the commercial and buys the product. ______

b) Estimate the probability that a consumer

1. buys the product if he sees the commercial. ______
2. does not buy the product if he does not see the commercial. ______

23. A study on the students in a class divided them according to gender and whether they are right- or left-handed.

Gender \ Handedness	Right-handed	Left-handed	Total
Boy	14	4	
Girl	10	2	
Total			

a) What is the probability that a student from this class is

1. left-handed? ______
2. is a right-handed boy? ______ 3. is a left-handed girl? ______

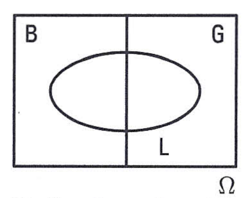
b) What is the probability that the student is left-handed if it is

1. a boy? ______ 2. a girl? ______

c) What is the probability that a student is a boy if the student is

1. right-handed? ______ 2. left-handed? ______

24. In class of 30 students, there are 20 boys. We know that 5 students wear glasses, of which 3 are girls.



B: "to be a boy".
G: "to be a girl".
L: "to wear glasses".

a) Represent the situation by a Venn diagram.

b) What is the probability that a student chosen at random in the class is

1. a boy and wears glasses? ______
2. a girl and does not wear glasses? ______
3. a boy if the student wears glasses? ______

25. A study of the last 50 patients who went to a dentist shows that 10 patients were treated for a cavity, 16 patients had their teeth cleaned and 4 patients had both treatments. Estimate the probability that the dentist's next patient

a) has at least one of these two treatments. ______

b) has a treatment for a cavity only. ______

c) has only one of these two treatments. ______

d) has neither of these two treatments. ______

26. A study of drivers summoned to the municipal court for traffic law violations shows that

- 75% of the drivers are actually guilty;
- 60% of the drivers are convicted for the violation;
- 50% of the drivers are actually guilty and convicted for the violation.

Calculate the probability that a driver is

a) not guilty.

b) guilty and not convicted for the violation.

c) convicted and not guilty.

d) not guilty and not convicted for the violation.

27. A cosmopolitan neighbourhood on the island of Montreal has a population of 2400 people. Among these, 1500 people speak French, 800 people speak English and 600 speak both languages. What is the probability that a person chosen at random from that neighbourhood

a) speaks French?

b) speaks English only?

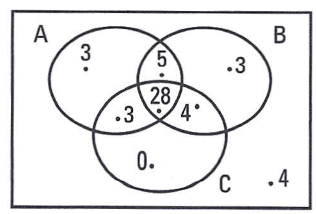
c) speaks at least one of these two languages?

d) speaks only one of these two languages?

e) speaks English if she speaks French?

f) speaks French if she speaks English?

28. Of the 50 students who wrote the English, biology and chemistry exams, 28 students passed all three, 3 students passed in English only, 3 students passed in biology only and no student passed in chemistry only. Moreover, 3 students passed in English and chemistry but failed in biology; 5 students passed in English and biology but failed in chemistry and 4 students passed in biology and chemistry but failed in English. A student is chosen at random. What is the probability that this student



a) passed in English?

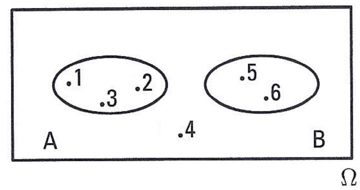
b) failed all three exams?

c) passed at least one of the exams?

5.6 Axioms and theorems

ACTIVITY 1 Axioms for calculating probabilities

A die is rolled once and the 6 possible outcomes are **equally likely**.
Consider the following incompatible events,
A: "rolling a number less than 4" and
B: "rolling a number greater than 4".
Using events A and B, verify the following three axioms.



Axiom 1: For any event E, we have: $0 \leq P(E) \leq 1$.

____

Axiom 2: $P(\Omega) = 1$

____

Axiom 3: If A and B are two incompatible events, $P(A \cup B) = P(A) + P(B)$.

____

____

These three axioms remain valid when the possible outcomes of a random experiment are not equally likely.

AXIOMS FOR CALCULATING PROBABILITIES

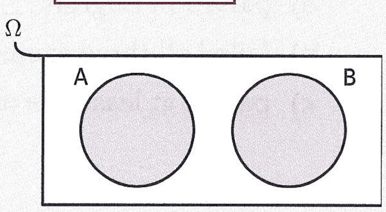
Let $\Omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$ be the universal set associated with a random experiment where the possible outcomes $\omega_1, \omega_2, \ldots, \omega_n$ are **not necessarily equally likely**.

Axiom 1: For any event E, we have: $0 \leq P(E) \leq 1$

Axiom 2: The probability that the certain event occurs is equal to 1. $P(\Omega) = 1$

Axiom 3: If A and B are two **mutually exclusive** events,

$$P(A \cup B) = P(A) + P(B)$$



Axiom 4: If $A_1, A_2, \ldots$ is a **sequence of mutually exclusive events**, we have:

$$P(A_1 \cup A_2 \cup \ldots) = P(A_1) + P(A_2) + \ldots$$

As a consequence of these axioms:
For any outcome ω, the probability $P(\omega)$ is **positive**. $P(\omega) > 0$

The **sum** of the probabilities of the possible outcomes is equal to 1. $P(\omega_1) + P(\omega_2) + \ldots + P(\omega_n) = 1$

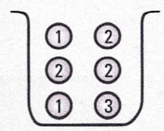
A consequence of these axioms is the following theorem which enables us to calculate the probability that an event occurs when the possible outcomes are not necessarily equally likely.

PROBABILITY OF AN EVENT

If A is any event, the probability P(A) that event A occurs is equal to the sum of the individual probabilities of the elements in A.

$$\text{If } A = \{a_1, a_2, \ldots, a_m\} \text{ then } P(A) = P(a_1) + P(a_2) + \ldots + P(a_m)$$

Ex.: Consider the jar on the right. A ball is drawn at random from the jar and its number is noted.



We have $\Omega = \{1, 2, 3\}$; $P(1) = \frac{2}{6}$; $P(2) = \frac{3}{6}$; $P(3) = \frac{1}{6}$.

The possible outcomes 1, 2 and 3 are not equally likely.

Let A be the event "drawing an odd number".

We have: $A = \{1, 3\}$ and $P(A) = P(1) + P(3) = \frac{3}{6}$.

1. Three horses H_1, H_3 and H_3 compete in a race. H_1 has twice as many chances as H_2 of winning and H_2 has twice as many chances as H_3 of winning. The experiment consists of observing the winner of the race.

a) Describe extensively the universal set Ω. ______

b) If x represents the probability that H_3 wins, express in terms of x

1. the probability that H_2 wins. ______ 2. the probability that H_1 wins. ______

c) Determine the respective probabilities $P(H_1)$, $P(H_2)$ and $P(H_3)$ of each of the three horses winning.

______

2. A die is loaded so that when it is rolled, each even number has twice as many chances of showing than each odd number.

a) Let x be the probability that a 1 is rolled.

1. Complete the table on the right.
2. Determine the value of x.

ω	1	2	3	4	5	6
P(ω)	x					

______

3. Determine the probability of each outcome.

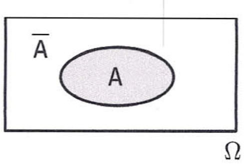
ω	1	2	3	4	5	6
P(ω)						

b) Calculate the probability of the following events.

1. "Rolling an even number." ______ 2. "Rolling a number less than 4." ______

ACTIVITY 2 Probability of the contrary event

a) A die is rolled once. Consider the event A: "rolling a number greater than or equal to 5".



1. Describe in words the event $\overline{A}$.

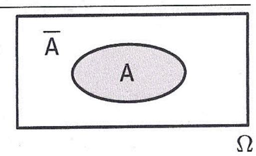
______

2. After describing extensively the events A and $\overline{A}$ calculate

1) $P(A)$. ______________________ 2) $P(\overline{A})$. ______________________

3. Verify that $P(\overline{A}) = 1 - P(A)$. ______________________

b) Justify the steps showing that $P(\overline{A}) = 1 - P(A)$.

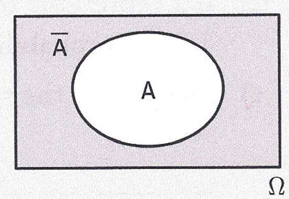


Statements	Justifications
1. $P(A \cup \overline{A}) = P(\Omega)$	
2. $P(A \cup \overline{A}) = P(A) + P(\overline{A})$	
3. $P(\Omega) = 1$	
4. $P(A) + P(\overline{A}) = 1$	
5. $P(\overline{A}) = 1 - P(A)$	

PROBABILITY OF THE CONTRARY EVENT

For any event A, we have

$$\boxed{P(\overline{A}) = 1 - P(A)}$$



Ex.: Let $\Omega = \{1, 2, \ldots, 6\}$ be the universal set associated with the experiment consisting of rolling a fair die once.

Let A: "rolling a number less than or equal to 2" $A = \{1, 2\}$; $P(A) = \frac{2}{6}$

and $\overline{A}$: "rolling a number greater than 2". $\overline{A} = \{3, 4, 5, 6\}$; $P(\overline{A}) = \frac{4}{6}$

We have: $P(\overline{A}) = 1 - P(A)$.

ACTIVITY 3 Probability of the impossible event

Justify the steps showing that $P(\varnothing) = 0$.

Statements	Justifications
1. $\varnothing = \overline{\Omega}$	
2. $P(\overline{\Omega}) = 1 - P(\Omega)$	
3. $P(\overline{\Omega}) = 1 - 1 = 0$	
4. $P(\varnothing) = 0$	

PROBABILITY OF THE IMPOSSIBLE EVENT

The probability that the **impossible** event occurs is zero.

$$P(\varnothing) = 0$$

ACTIVITY 4 Probability of the event "A or B"

Consider the random experiment consisting of rolling a die once.
Let A and B be the events A: "rolling a number greater than or equal to 5" and B: "rolling an even number".

a) Describe extensively the following events.

1. $A =$ ______________ 2. $B =$ ______________
3. $A \cap B =$ ______________ 4. $A \cup B =$ ______________

b) Calculate the following probabilities.

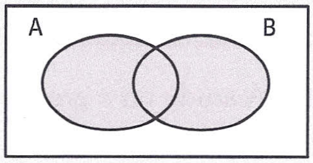
1. $P(A) =$ ______________ 2. $P(B) =$ ______________
3. $P(A \cap B) =$ ______________ 4. $P(A \cup B) =$ ______________

c) Verify that $P(A \cup B) = (P(A) + P(B) - P(A \cap B)$. ______________

PROBABILITY OF THE UNION

- Let A and B be two events associated with a random experiment. We have:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



Ex.: A card is drawn from a 52-card deck. Calculate the probability of drawing a queen or a heart.
Let A: "drawing a queen" and B: "drawing a heart".
We have $A \cup B$: "drawing a queen or a heart" and $A \cap B$: "drawing the queen of hearts".

$$P(A) = \frac{4}{52}, P(B) = \frac{13}{52}, P(A \cap B) = \frac{1}{52}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$$

ACTIVITY 5 Probability of the difference event

Consider the random experiment consisting of rolling a die once.
Let A and B be the events A: "rolling an even number" and B: "rolling a number less than 5".

a) Describe extensively the following events.

1. $A =$ ______________ 2. $B =$ ______________ 3. $A \cap B =$ ______________
4. $A \cap \overline{B} =$ ______________ 5. $P(A \cap \overline{B}) =$ ______________

b) Calculate the following probabilities.

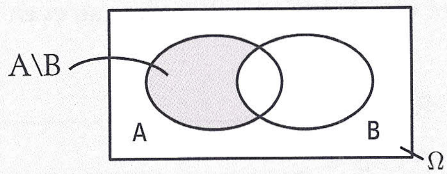
1. $P(A) =$ ____ 2. $P(B) =$ ____ 3. $P(A \cap B) =$ ____ 4. $P(A \cap \overline{B}) =$ ____ 5. $P(\overline{A} \cap B) =$ ____

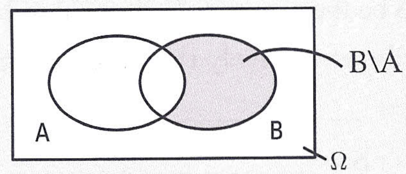
c) Verify that

1. $P(A \cap \overline{B}) = P(A) - P(A \cap B)$. ____________________
2. $P(\overline{A} \cap B) = P(B) - P(A \cap B)$. ____________________

PROBABILITY OF THE DIFFERENCE

- Let A and B be two events associated with a random experiment. We have:



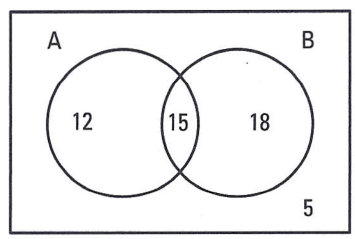


$P(A \backslash B) = P(A) - P(A \cap B)$ $P(B \backslash A) = P(B) - P(A \cap B)$

Ex.: A card is drawn from a 52-card deck.
Let A and B be the events A: "drawing a queen" and B: " drawing a heart".
$P(A) = \frac{4}{52}$; $P(B) = \frac{13}{52}$; $P(A \cap B) = \frac{1}{52}$
A\B: "drawing a queen that is not a heart".
B\A: "drawing a heart that is not a queen".
$P(A \backslash B) = P(A) - P(A \cap B) = \frac{3}{52}$; $P(B \backslash A) = P(B) - P(A \cap B) = \frac{12}{52}$

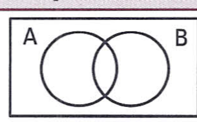
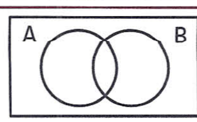
3. A study on a group of 50 students who wrote English and biology exams shows that

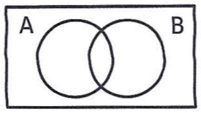
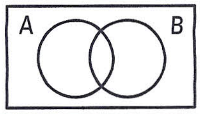
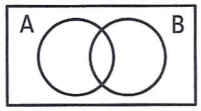
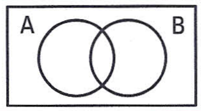
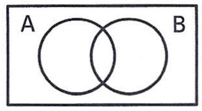
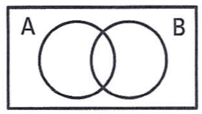
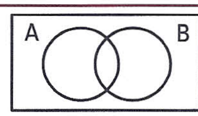
- 15 students passed both exams;
- 12 students passed the English exam but failed the biology one;
- 18 students passed the biology exam but failed the English one;
- 5 failed both exams.



A student is chosen at random from the group. Consider the following events:
A: "passing the English exam"; B: "passing the biology exam".

a) Complete the following table.

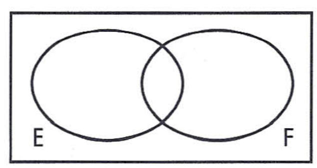
	Event	Representation	Interpretation	Probability
1.	A			
2.	B			

3.	$A \cap B$			
4.	$A \cup B$			
5.	$\overline{A}$			
6.	$\overline{A} \cap B$			
7.	$A \cap \overline{B}$			
8.	$\overline{A} \cap \overline{B} = \overline{A \cup B}$			
9.	$\overline{A} \cup \overline{B} = \overline{A \cap B}$			

b) Verify that

1. $P(\overline{A}) = 1 - P(A)$. ______________________
2. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. ______________________
3. $P(A \cap \overline{B}) = P(A) - P(A \cap B)$. ______________________
4. $P(\overline{A} \cap B) = P(B) - P(A \cap B)$. ______________________
5. $P(\overline{A} \cap \overline{B}) = 1 - P(A \cup B)$. ______________________
6. $P(\overline{A} \cup \overline{B}) = 1 - P(A \cap B)$. ______________________

4. A study shows that, in a group of 50 tourists, 28 speak English, 20 speak French and 10 speak English and French. A tourist is chosen at random from the group. Consider the following events:
E: "speaking English"; F: "speaking French".



a) Represent the situation on the Venn diagram on the right.

b) Write symbolically the following events and then calculate the probability of each of them.

1. The tourist speaks English. ______________________
2. The tourist speaks English but not French. ______________________
3. The tourist speaks English and French. ______________________
4. The tourist speaks English or French. ______________________
5. The tourist speaks neither English nor French. ______________________
6. The tourist does not speak English or does not speak French. ______________________

5. Let A and B be two events such that $P(A) = 0.7$; $P(B) = 0.6$; $P(A \cap B) = 0.5$. Calculate the following probabilities.

a) $P(\overline{A}) =$ ____________
b) $P(A \cup B) =$ ____________
c) $P(A \cap \overline{B}) =$ ____________
d) $P(\overline{A \cup B}) =$ ____________
e) $P(\overline{A} \cup \overline{B}) =$ ____________
f) $P(\overline{A \cap B}) =$ ____________
g) $P(\overline{A} \cap \overline{B}) =$ ____________
h) $P(\overline{A} \cap B) =$ ____________

6. A study on drivers summoned to the municipal court for traffic law violations shows that

- 75% of the drivers are actually guilty;
- 60% of the drivers are convicted for the violation;
- 50% of the drivers are actually guilty and convicted for the violation.

A driver appears before the judge. Consider the following events:
A: "the driver is actually guilty"; B: "the driver is convicted for the violation".

Calculate and interpret the following probabilities:

a) $P(\overline{A})$ ____________

b) $P(A \cup B)$ ____________

c) $P(A \cap \overline{B})$ ____________

d) $P(\overline{A} \cap B)$ ____________

e) $P(\overline{A} \cup \overline{B})$ ____________

7. An economist makes the following predictions for next week.

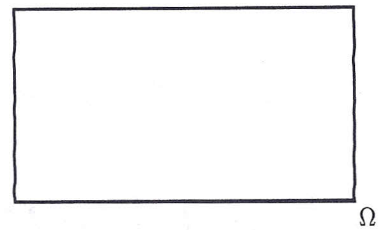
- There is a 40 out of 100 chance that the dollar will fall.
- There is a 30 out of 100 chance that interest rates will fall.
- There is a 20 out of 100 chance that both the dollar and interest rates will fall.

Calculate the probability that next week

a) the dollar or interest rates will fall. ____________
b) the dollar will fall and interest rates will not fall. ____________
c) the dollar and interest rates will not fall. ____________
d) the dollar or interest rates will not fall. ____________

8. A die is rolled once. Consider the following events.

A: " rolling a number less than 3" and
B: "rolling a number less than 5".



a) Describe extensively events A and B, represent them on the Venn diagram on the right and verify that $A \subseteq B$.

____

b) If event A occurs, can you conclude that event B occurs? ____________________

c) What can you conclude concerning the occurrence of event A when you know that event B does not occur?

____

d) Verify the property: $A \subseteq B \Rightarrow P(A) \leqslant P(B)$.

____

e) Calculate

1. $P(A \cap B)$. ________________ 2. $P(A \cup B)$. ________________

9. A die is rolled twice. Calculate the probability of the following events.

a) "We get a 6 on the 1st or 2nd roll". ____________________

b) "The sum of the numbers is equal to 7". ____________________

c) "The sum of the numbers is equal to 7 or we get an even number on the 1st roll." ______

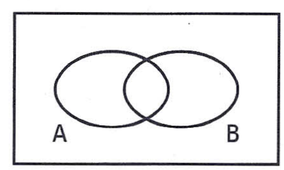
d) "We get the same number on each roll". ____________________

e) "The sum of the numbers is equal to 6 and we get a 6 on the 1st roll". __________

f) "We get an even or an odd number on the 1st roll". ____________________

10. A meteorologist makes the following forecast for tomorrow.

- There is a 50 out of 100 chance that the temperature will be below 0 °C.
- There is a 60 out of 100 chance that it will snow.
- There is a 30 out of 100 chance that the temperature will drop below 0 °C and that it will not snow.



A: "the temperature will be below 0°"
B: "it will snow"

What is the probability that tomorrow

a) the temperature is below 0 °C and it snows? ____________________

b) it snows and the temperature is not below 0 °C? ____________________

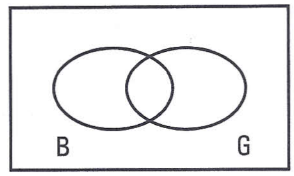
c) it does not snow and the temperature is not below 0 °C? ____________________

d) the temperature is below 0 °C or it snows? ____________________

5.7 Conditional probability

ACTIVITY 1 Conditional probability (equally likely outcomes situation)

In a group of 30 students, there are 18 boys. We observe a total of 3 boys and 2 girls wearing glasses. A student is chosen at random from the group.



Ω

B: "the student is a boy"

G: "the student wears glasses"

a) If the student is a boy, what is the probability that he wears glasses?

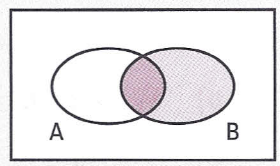
____

b) If the chosen student wears glasses, what is the probability that it is a boy? __________________________________

CONDITIONAL PROBABILITY (equally likely outcomes situation)

Let Ω be the universal set associated with a random experiment and B an event such that all outcomes favourable to B are **equally likely**.

The probability that an event A occurs given that the event B has occurred, written P(A|B), is defined by

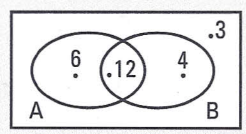


Ω

$$P(A \mid B) = \frac{\text{number of outcomes favourable to A among those favourable to B}}{\text{number of outcomes favourable to B}} = \frac{n(A \cap B)}{n(B)}$$

Ex.: In a group of 25 students, we observe that

- 18 pass the English test;
- 16 pass the biology test;
- 12 pass both tests.



Ω

The probability that a student passes in English if he passes in biology is written P(A|B).

We have: $P(A|B) = \frac{n(A \cap B)}{n(B)} = \frac{12}{16} = 75\ \%$.

The probability that a student passes in biology if he passes in English is written P(B|A).

We have $P(B|A) = \frac{n(A \cap B)}{n(A)} = \frac{12}{18} = 66.\overline{6}\ \%$.

1. Canadian tourists coming back from South America were interviewed. Among the 40 tourists in the group

- 20 tourists visited Argentina;
- 30 tourists visited Brazil;
- 12 tourists visited both these countries.

What is the probability that a tourist visited

a) Argentina if he visited Brazil? __________ **b)** Brazil if he visited Argentina? __________

c) Argentina if he did not visit Brazil? _____ **d)** Brazil if he did not visit Argentina? _____

2. The human resources director of a large company sorts 1000 employee files according to age and gender.

Age \ Gender	Male (M)	Female (F)	Total
under 30 years old (A)	100	150	250
30 to 40 years old (B)	240	210	450
over 40 years old (C)	180	120	300
Total	520	480	1000

a) A file is selected at random. What is the probability that the employee

1. is a male? ____________
2. is under 30 years old? ____________
3. is a male under 30? ____________

b) What is the probability that the file is that of an employee who

1. is under 30 years old, knowing that the employee is a male? ____________
2. is a male, knowing that the employee is under 30? ____________

c) Interpret the following expressions and then calculate the probabilities.

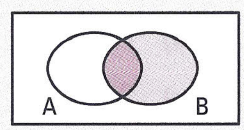
1. P(F) ____________
2. P(F|A) ____________
3. P(A|F) ____________
4. P(M|C) ____________
5. P(F|A ∪ B) ____________
6. P(M|B ∪ C) ____________

CONDITIONAL PROBABILITY

In general, given two events A and B,

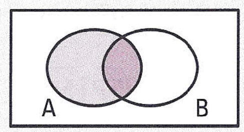
– the probability of A, knowing B, is defined by:

$$P(A\,|\,B) = \frac{P(A \cap B)}{P(B)} \quad (P(B) \neq 0)$$



– the probability of B, knowing A, is defined by:

$$P(B\,|\,A) = \frac{P(A \cap B)}{P(A)} \quad (P(A) \neq 0)$$



Ex.: At a college,

- 60% of students pass the English exam (P(A) = 0.6);
- 50% of students pass the biology exam (P(B) = 0.5);
- 40% of students pass both exams (P(A ∩ B) = 0.40).

The probability that a student passes in English, if he passed in biology, is equal to:

$$P(A\,|\,B) = \frac{P(A \cap B)}{P(B)} = \frac{0.40}{0.50} = \frac{4}{5}.$$

The probability that a student passes in biology, if he passed in English, is equal to:

$$P(B\,|\,A) = \frac{P(A \cap B)}{P(A)} = \frac{0.40}{0.60} = \frac{2}{3}.$$

3. A fair die is rolled once. Consider the following events.
A: "rolling an even number" and B: "rolling a number less than 5".

a) Calculate.
1. $P(A)$ ______ 2. $P(B)$ ______ 3. $P(A \cap B)$ ______
4. $P(A|B)$ ______ 5. $P(B|A)$ ______

b) Verify.
1. $P(A|B) = \frac{P(A \cap B)}{P(B)}$ ______ 2. $P(B|A) = \frac{P(A \cap B)}{P(A)}$ ______

4. For their extracurricular activities, 20% of the students at a school sing in the choir, 15% do theatre and 5% participate in both these activities. What is the probability that a student

a) sings in the choir if he does theatre? ______

b) does theatre if he sings in the choir? ______

5. A study done on secondary 5 students shows that
- 70% of students pass their mathematics course;
- 60% of students pass their physics course;
- 50% of students pass both courses.

A student is chosen at random. Consider the following events.
M: "the student passes in mathematics"; P: "the student passes in physics".
Calculate the following probabilities and interpret the results.

a) $P(M|P)$ ______

b) $P(\overline{M}|P)$ ______

c) $P(M|\overline{P})$ ______

d) $P(\overline{M}|\overline{P})$ ______

e) $P(P|M)$ ______

f) $P(\overline{P}|M)$ ______

g) $P(\overline{P}|\overline{M})$ ______

h) $P(P|\overline{M})$ ______

6. In a survey, 40% of people said they read weekly magazines, 50% read monthly magazines and 20% reads both types of magazines. What is the probability that one of the surveyed individual

a) reads monthly magazines if he reads weekly magazines? ______

b) reads weekly magazines if he reads monthly magazines? ______

7. a) In a random experiment, two events A and B such that $A \subseteq B$ are considered. What can be said about $P(B|A)$?

Justify your answer. ______________________________

b) A card is drawn from a 52-card deck. Consider the events A: "drawing a queen" and B: "drawing a face card".

1. What can be said about the events A and B? ______________________________
2. Calculate $P(B|A)$. ______________________________
3. Calculate $P(A|B)$. ______________________________

ACTIVITY 2 Probability of the event "A and B"

a) Let A and B be any two events associated with a random experiment and such that $P(A) \neq 0$ and $P(B) \neq 0$. Show that

1. $P(A \cap B) = P(A) \times P(B|A)$.

____

2. $P(A \cap B) = P(B) \times P(A|B)$.

____

b) In a company, 40% of employees are men and 20% of men are Anglophone. An employee is chosen at random. Consider the following events:

M: "the employee is a man", A: "the employee is Anglophone".

Calculate the probability that the chosen employee is an Anglophone man.

____

PROBABILITY OF THE INTERSECTION

Let A and B be two events associated with a random experiment and such that $P(A) \neq 0$ and $P(B) \neq 0$.

We have: $P(A \cap B) = P(A) \times P(B|A)$ or $P(A \cap B) = P(B) \times P(A|B)$

Ex.: In a factory where bolts are made, a machine M provides 40% of the total production. A study showed that 10 % of the bolts produced by machine M are defective. A bolt is chosen at random from the total production. What is the probability that the bolt comes from machine M and is defective?

Consider the events M: "the bolt comes from machine M" and D: "the bolt is defective".

We have: $P(M) = 0.40$; $P(D|M) = 0.10$. We seek $P(M \cap D)$.

$P(M \cap D) = P(M) \times P(D|M) = 0.40 \times 0.10 = 0.04$

8. In a class, 40% of students are boys. In addition, 20% of boys have blue eyes while 25% of girls have blue eyes. A student is chosen at random in this class. What is the probability that the chosen student is

a) a boy with blue eyes? ____________ **b)** a girl with blue eyes? ____________

c) a boy who doesn't have blue eyes? ______ **d)** a girl who doesn't have blue eyes? ______

ACTIVITY 3 Tree

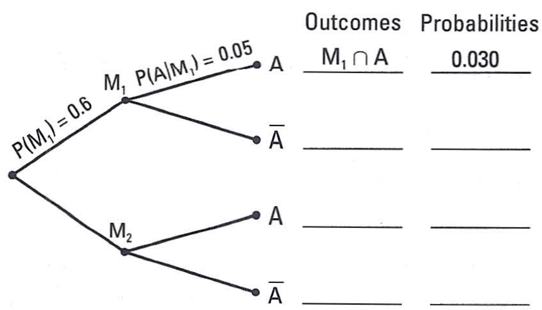
In a factory that makes bolts, production is done by two machines, M_1 and M_2.
M_1 provides 60% of the total production and M_2 provides 40 %.
Among the bolts produced by M_1, 5% are defective, while 3% of those produced by M_2 are.
A bolt is chosen at random from the total production.
First, its origin is examined: M_1 or M_2, then its state: defective (A) or non defective ($\bar{A}$).

a) Calculate the following probabilities and interpret the results.

1. $P(M_1)$ ____________________
2. $P(M_2)$ ____________________
3. $P(A|M_1)$ ____________________
4. $P(A|M_2)$ ____________________
5. $P(M_1 \cap A)$ ____________________
6. $P(M_2 \cap A)$ ____________________

b) Complete the tree below by indicating the probabilities on each branch, then determine the probability of each outcome. (Note: the outcome $M_1 \cap A$ can be written: (M_1, A)).

The 1st step consists of noting the origin of the bolt and the 2nd consists of indicating the state of the bolt.



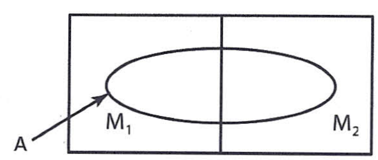
c) Complete the extensive description of the set Ω of possible outcomes.

$\Omega = \{(M_1, A),$ ____________________ $\}$

d) Verify that the sum of the probabilities of the possible outcomes is equal to 1.

____

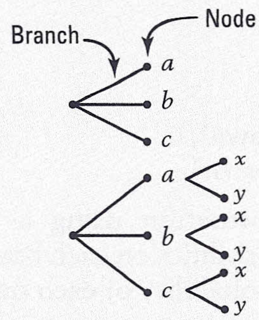
e) Write the probability of each possible outcome on the Venn diagram on the right.



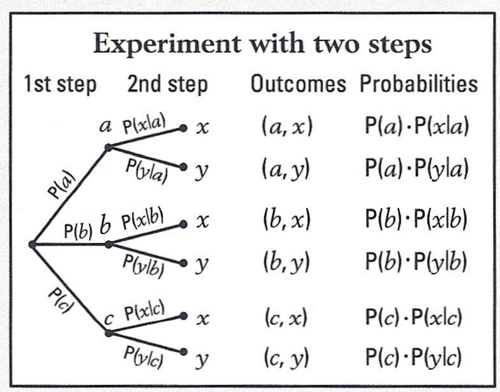
TREE

The outcomes of a random experiment having many steps can be determined using a **tree** or **rooted tree diagram**.

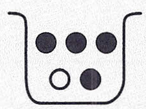
- For the **first step**, draw as many **branches** as there are possible outcomes for the first step.
- For the **second step**, draw, starting from each node of the first step, as many branches as there are possible outcomes for the second step.
- Indicate the possible outcomes of the experiment.
- On each branch, indicate the probability.
- Deduce the probability of each outcome by calculating the product of the probabilities indicated on the branches giving rise to this outcome.



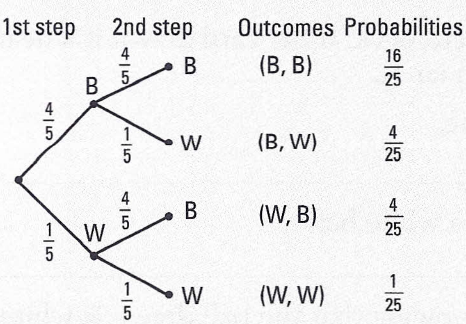
The first step has three branches. At each node, the outcome is indicated.



Ex.: Two balls are drawn successively from a jar containing four black balls and one white ball.

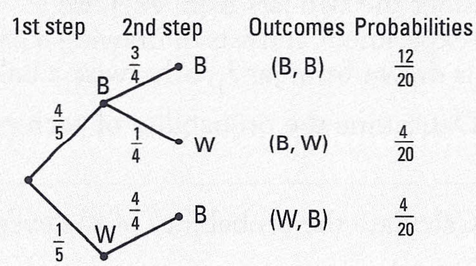


First situation: draw **with replacement**



$\Omega = \{(B, B), (B, W), (W, B), (W, W)\}$

Second situation: draw **without replacement**



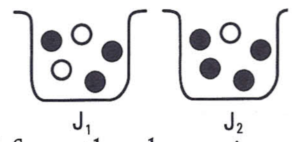
$\Omega = \{(B, B), (B, W), (W, B)\}$

The event E: "drawing a total of one white ball" is described extensively by $E = \{(W, B), (B, W)\}$.

The probability that event E occurs is

- draw with replacement: $P(E) = P(W, B) + P(B, W) = \frac{4}{25} + \frac{4}{25} = \frac{8}{25}$.
- draw without replacement: $P(E) = P(W, B) + P(B, W) = \frac{4}{20} + \frac{4}{20} = \frac{8}{20}$.

9. Consider two jars.



Jar J_1 contains three black balls and two white balls;
Jar J_2 contains four black balls and one white ball.
The experiment consists of choosing a jar at random, then drawing a ball from the chosen jar.

Consider the events:
J_1: "jar J_1 is chosen";
J_2: "jar J_2 is chosen";
B: "a black ball is drawn";
W: "a white ball is drawn".

a) Represent the situation using a tree and indicate the probabilities on each branch, then determine the probability of each outcome.

b) Determine the set Ω of possible outcomes.

____

c) Verify that the sum of the probabilities of the possible outcomes is equal to 1. ______________________________

d) Consider the event W: "drawing a white ball".
1. Describe event W extensively. ______________________________
2. Calculate P(W). ______________________________

e) Determine the probability that jar J_1 was chosen knowing that the ball drawn is white.

____

10. Consider the two jars from exercise 9.
The experiment consists of drawing a card from a 52-card deck. If the card drawn is a heart, a ball is drawn from jar J_1, otherwise a ball is drawn from jar J_2.

a) Determine the probability of each possible outcome.

____

b) Calculate the probability of the event W: "drawing a white ball".

____

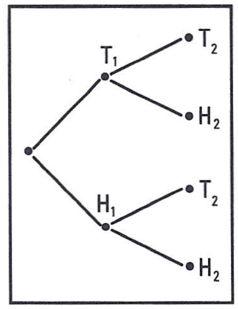
c) Determine the probability that jar J_1 was chosen knowing that the ball drawn is white.

____

5.8 Independent events

ACTIVITY 1 Independent events

A coin is tossed twice. Consider the following events.
T_i: "getting tails on the ith toss", $i = 1, 2$.
H_i: "getting heads on the ith toss", $i = 1, 2$.



a) Determine.

1. $P(T_1)$ ______________ 2. $P(T_2)$ ______________

3. $P(T_2 | T_1)$ ______________ 4. $P(T_2 | H_1)$ ______________

b) Answer true or false.

1. $P(T_2 | T_1) = P(T_2)$ ______________ 2. $P(T_2 | H_1) = P(T_2)$ ______________

c) Does the probability of getting tails on the 2nd toss depend on the outcome of the 1st toss?

____

d) 1. Describe in words the event $T_1 \cap T_2$ then calculate $P(T_1 \cap T_2)$.

____

2. Verify that $P(T_1 \cap T_2) = P(T_1) \times P(T_2)$. ______________

INDEPENDENT EVENTS

- Two events A and B are **independent** if
  - the probability that event A occurs is not influenced by the fact that event B occurred.

$$P(A | B) = P(A)$$

  - the probability that event B occurs is not influenced by the fact that event A occurred.

$$P(B | A) = P(B)$$

- Two events A and B that are not independent are called **dependent**.
  Ex.: A jar contains red balls and black balls.
  - When two balls are drawn **with replacement**, the event R_2: "getting a red ball on the second draw" is **independent** from the event R_1: "getting a red ball on the first draw". We have: $P(R_2) = P(R_2 | R_1)$. The draws are called **independent**.
  - When two balls are drawn **without replacement**, events R_1 and R_2 are **dependent**. We have: $P(R_2) \neq P(R_2 | R_1)$. The draws are called **dependent**.
- **Theorem**: Let A and B be two events associated with a random experiment. We have:

$$\text{A and B are independent if, and only if, } P(A \cap B) = P(A) \times P(B)$$

Ex.: A die is rolled at the same time a coin is tossed.
The events A: "the die shows 6" and B: "the coin shows tails" are independent.
We have: $P(A \cap B) = P(A) \cdot P(B) = \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$.
Thus, there is a one out of 12 chance that the die shows 6 and the coin shows tails.

1. Complete using the appropriate symbol = or ≠.
Two events A and B are dependent if
1. $P(A|B)$ ____ $P(A)$. 2. $P(B|A)$ ____ $P(B)$. 3. $P(A \cap B)$ ____ $P(A) \times P(B)$.

2. A die is rolled twice. Consider the following events.
A: "getting 6 on the 1st roll", B: "getting 6 on the 2nd roll" and C: "rolling a sum equal to 10".

a) Are events A and B independent? Justify your answer.

____

____

b) Are events A and C dependent? Justify your answer.

____

____

3. Consider, in a family with 3 children, the following events.
A: "the oldest child is a boy", B: "all the children are of same gender" and C: "there is a total of two boys in the family".

a) Determine if events A and B are independent.

____

____

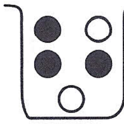
____

b) Determine if events A and C are independent.

____

____

4. A jar contains 2 white balls and 3 black balls.
Three balls are drawn successively **without replacement**.
Consider the following events.
W_i: "getting a white ball on the ith draw", $i = 1, 2, 3$.
B_i: "getting a black ball on the ith draw", $i = 1, 2, 3$.
Calculate and interpret the outcomes



a) $P(B_1)$ ____

b) $P(B_2|B_1)$ ____

____

c) $P(B_3|B_1 \cap B_2)$ ____

____

d) $P(B_1 \cap B_2)$ ____

____

5. Three balls are drawn successively **with replacement** from the jar of exercise 4. Calculate.

a) $P(W_1)$ ______________________

b) $P(W_2 | W_1)$ ______________________

c) $P(W_1 \cap W_2)$ ______________________

d) $P(W_3 | W_1 \cap W_2)$ ______________________

e) $P(W_1 \cap W_2 \cap W_3)$ ______________________

SEQUENCE OF INDEPENDENT EVENTS

Let $A_1, A_2, \ldots, A_n$, be n events associated with a random experiment.

$A_1, A_2, \ldots, A_n$ are independent if, and only if, $P(A_1, \cap A_2 \cap \ldots A_n) = P(A_1) \times P(A_2) \times \ldots P(A_n)$

Ex.: A die is rolled three times. Consider the following events.

A_i: "getting a 6 on the ith roll", $i = 1, 2, 3$.

– A_1, A_2 and A_3 are independent and $P(A_i) = \frac{1}{6}$.

– $P(A_1 \cap A_2 \cap A_3) = P(A_1) \times P(A_2) \times P(A_3) = \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216}$

– There is a one out of 216 chance of getting a 6 on each roll.

6. At archery, Riva hits the target 3 times out of 5. She shoots three times at the target. Calculate the probability of the following events.

a) She hits the target on the 1st shot, misses on the second and hits the target on the third.

____

b) She hits the target on each shot. ______________________

c) She misses the target on each shot. ______________________

d) She hits the target only once. ______________________

7. Four cards are drawn successively from a 52-card deck. Calculate the probability of getting, on each draw, a heart when the draw is done

a) with replacement. ______________________

b) without replacement. ______________________

8. Nathalie and Eric write a mathematics exam. Nathalie has a 7 out of 10 chance to pass the exam while Eric has a 6 out of 10 chance.

Calculate the probability of the following events.

a) "Both Nathalie and Eric pass." ______________________

b) "Nathalie passes and Eric fails." ______________________

c) "Nathalie fails and Eric passes." ______________________

d) "Both Nathalie and Eric fail." ______________________

e) "Only one of the two passes." ______________________

f) "At least one of the two passes." ______________________

5.9 Random variable

ACTIVITY 1 Random variable

Consider the universal set Ω describing the possible outcomes of the random experiment consisting of tossing a coin three times.
$\Omega = \{(T,T,T), (T,T,H), (T,H,T), (T,H,H), (H,T,T), (H,T,H), (H,H,T), (H,H,H)\}$
Let X represent the variable which associates each possible outcome with the total number of tails obtained.

ω	x
(T, T, T)	
(T, T, H)	
(T, H, T)	
(T, H, H)	
(H, T, T)	
(H, T, H)	
(H, H, T)	
(H, H, H)	

a) Complete the table on the right which associates each possible outcome with the value x taken by X.

b) Let D_X represent the set of possible values taken by X. Describe D_X extensively. ______

c) Are the eight possible outcomes of the experiment equally likely? ______

d) Calculate the following probabilities.

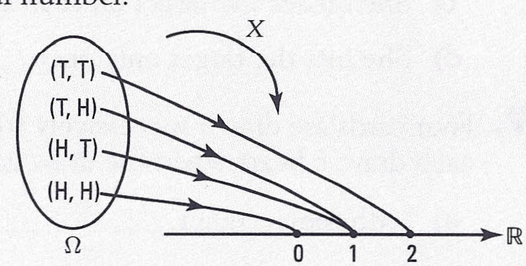
1. $P(X = 0) =$ ____ 2. $P(X = 1) =$ ____ 3. $P(X = 2) =$ ____ 4. $P(X = 3) =$ ____

e) Verify that 1. $P(X = x) > 0$ if $x \in D_X$.
2. $P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) = 1$.

RANDOM VARIABLE

- Given a random experiment and the corresponding universal set Ω, a **random variable** associates each outcome of the experiment with a real number.

  Ex.: $\Omega = \{(T, T), (T, H), (H, T), (H, H)\}$ is the universal set of the random experiment consisting of tossing a coin twice; X represents the random variable which associates each outcome with the number of tails.



- The set, written D_X, of values taken by a random variable X is called its **domain** or **support**.

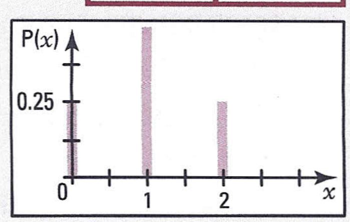
  We have: $D_X = \{0, 1, 2\}$.

- The **probability** that a variable X takes the value x is written $P(X = x)$, $P_X(x)$ or $P(x)$.

- The set of couples $\left\{\left(0, \frac{1}{4}\right), \left(1, \frac{1}{2}\right), \left(2, \frac{1}{4}\right)\right\}$ represents the **probability distribution of the random variable X**. This distribution gives the possible values for the random variable and the way the probabilities are spread out. This distribution is often expressed as a table (called **distribution table** or **probability table**) such as the one on the right.

Distribution of X

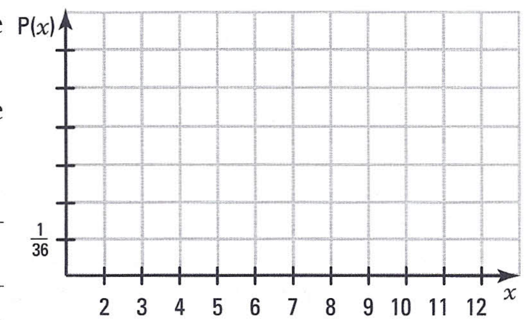
X	$P(X = x)$
0	0.25
1	0.50
2	0.25



- Let $D_X = \{x_1, x_2, \ldots, x_n\}$ be the domain of a random variable X. We have the following properties:
  - $P(X = x) > 0$ for all x in the domain D_X
  - $P(X = x_1) + P(X = x_2) + \ldots + P(X = x_n) = 1$ or $\sum P(x_i) = 1$

1. A die is rolled once. Let X represent the outcome. Determine the probability distribution of the random variable X. ______________________________

2. A die is rolled twice. Let X represent the sum of the numbers obtained.



a) Determine the probability distribution of the random variable X.

____

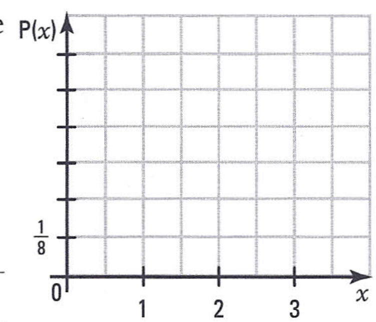
____

b) The mode of a random variable represents the most likely value for this variable.
What is the mode of X? ______________________________

c) Represent the distribution of X by a bar graph.

d) Verify, by observing the bar graph, that the distribution of X is symmetric and that the axis of symmetry is the vertical line passing through the mode.

3. Consider a family with three children. X is the random variable that gives the total number of boys in the family.



a) Determine the distribution of X.

b) Represent the distribution of X by a bar graph.

c) The distribution of X is bimodal. What are the two modes?

____

d) Is the distribution of X symmetric? ______________________________

4. A die is loaded so that, when it is rolled, each even face has twice as many chances of showing as each odd face. The random variable X represents the number rolled. Let p be the probability that a "1" is rolled.

a) Complete the probability distribution of the random variable X.

X						
P(x)						

b) Find the value of p. ______________________________

c) Calculate $P(x \leq 3)$. ______________________________

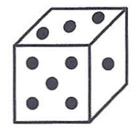
5.10 Expected value of a random variable

ACTIVITY 1 Expected value of a random variable

Consider the following game: "Roll a die and receive in dollars the number shown on the die when this number is even or lose in dollars the number shown if it is odd." Let X represent the player's gain.

a) Determine the distribution of X.

X						
$P(X = x)$						



b) For this game, a gain corresponds to a positive value of X; a loss (or negative gain), to a negative value of X. Show that it is equally probable to win than to lose at this game.

____

____

c) To determine if this game is favourable to the player, we must find his average gain. This average, called expected value of the gain (or expected gain), is calculated by weighing each possible gain (each possible value of the random variable X) by its probability.

1. Calculate the expected gain. __________ 2. Is the game favourable to the player? _____

A game is fair when the expected gain is zero. Though it is equally probable to win than to lose at this game (see b), this game is not fair since it is favourable to the player (see c).

EXPECTED VALUE OF A RANDOM VARIABLE

- Let X be a random variable whose probability distribution is:

X	x_1	x_2	…	x_i	…	x_n
$P(x)$	$p(x_1)$	$p(x_2)$		$p(x_i)$		$p(x_n)$

The **expected value** of X, written $E(X)$, also written symbolically μ (read mu) is defined by:

$$\mu = E(X) = x_1 p(x_1) + \ldots + x_n p(x_n) \quad \text{or} \quad \mu = E(X) = \sum x_i p(x_i)$$

Ex.: Consider the following game: "Toss a coin twice and lose \$1 if you get no tails, receive \$0 if you get exactly one tails or receive \$2 if you get exactly two tails." Let X represent the player's gain.

– The probability distribution of X is:

x	-1	0	2
$P(x)$	0.25	0.50	0.25

– We have: $\mu = E(X) = -1 \times 0.25 + 0 \times 0.50 + 2 \times 0.25 = \$0.25$.

In the context of a game, if the random variable X describes the gain, the expected value of X is called **expected gain**.
If $E(X) < 0$: the game is **unfavourable** to the player.
If $E(X) = 0$: the game is **fair**.
If $E(X) > 0$: the game is **favourable** to the player.

Thus, in this situation, the game is favourable to the player since the expected gain is positive ($E(X) = \$0.25$). On average, the player wins \$0.25 per game.

1. A fair die is rolled once. The random variable X represents the number shown on the die. Calculate $E(X)$.

2. A fair die is rolled twice.

a) Let X_1 represent the number obtained on the first roll and X_2 the number obtained on the second roll. The random variable X represents the sum of the numbers obtained.

1. Calculate $E(X_1)$.
2. Calculate $E(X_2)$.

b) Calculate $E(X)$.

3. **a)** A coin is tossed twice. Let X represent the total number of tails obtained. Calculate $E(X)$.

b) A coin is tossed three times. Let X represent the total number of tails obtained. Calculate $E(X)$.

c) A coin is tossed four times. Let X represent the total number of tails obtained. Calculate $E(X)$.

d) A coin is tossed n times. Let X represent the total number of tails obtained. Express $E(X)$ as a function of n.

4. What is the expected total number of girls in a family with:

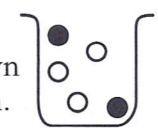
a) two children?

b) three children?

c) four children?

d) n children?

5. A jar contains two black balls and three white balls.
Consider the following game: "Draw a ball and receive $1 when the ball drawn is white or lose $2 when the ball drawn is black." Let X represent the player's gain.



a) Give the probability distribution of the random variable X.

b) What is the expected net gain?

c) Is the game favourable to the player? Justify your answer.

ACTIVITY 2 Transformation of a random variable

Consider the following game: roll a die once and receive in dollars twice the number rolled.
A player must play $7 to play this game.
Let X represent the number rolled and Y the player's net gain.

a) What is the player's net gain if he rolls a 5?

b) Express the net gain Y as a function of X.

c) Determine the probability distribution of X, then calculate $E(X)$.

x						
$P(x)$						

d) Deduce the probability distribution of Y, then calculate $E(Y)$.

y						
$P(y)$						

____

e) What can be said about this game? ______________________________

f) Verify the following propositions: If X and Y are two random variables such that $Y = aX + b$, where a and b are two real constants, then $E(Y) = a\,E(X) + b$. ______________________________

LINEAR TRANSFORMATION OF A RANDOM VARIABLE

- If X and Y are two random variables such that

  $Y = a\,X + b$ (a and b are real constants)

  then,

  $E(Y) = a\,E(X) + b$

6. Ten thousand lottery tickets were sold. There is one \$10 000 prize, ten \$1000 prizes and one hundred \$100 prizes. What is the expected net gain for this lottery if a ticket costs \$3.50?

____

7. Consider the following game: "Roll a die once and receive in dollars four times the number rolled." A player must pay \$15 to play this game.
Is the game favourable to the player? Justify your answer. ______________________________

8. Consider the following game: "Roll a die once and receive in dollars four times the number obtained on the die." How much should a player pay to play this game so that it is

a) fair? ______________ **b)** favourable to the player? ______________

9. Consider the following game: "Roll a die twice and receive in dollars the sum of the numbers obtained on the die." How much should a player pay to play this game so that it is

a) fair? ______________ **b)** favourable to the player? ______________

10. Consider the following game: "Toss a coin twice and lose \$10 if you get no tails, win \$4 if you get only one tails or win m dollars if you get two tails." Determine m so that this game is

a) fair? ______________ **b)** unfavourable to the player? ______________

11. Consider the following game: "Toss a coin three times and receive in dollars twice the number of tails obtained." How much should a player pay to play this game so that it is fair? __________

12. A roulette is divided into spaces numbered 1 to 10. If the number that comes out is even, you receive \$1; if the number that comes out is seven, you receive \$5. You receive nothing for the other numbers. How much should you pay to play this game so that is it fair? ______________

13. A firm is considering investing in a software company's stocks for a period of one year. The possible rates of return are described in the table on the right.

Rate of return	Probability
6 %	20 %
8 %	30 %
10 %	35 %
12 %	15 %

a) What would be the most likely rate of return? ___________

b) What would be the expected rate of return? ___________

14. An investor has to choose between the following two options:
Option A: invest for one year at a fixed interest rate of 8%;
Option B: invest in stocks of a company listed on the stock market with the possible rates of return given on the right.
Determine a and b so that both options are equivalent.

Rate of return	Probability
–10 %	10 %
0 %	40 %
10 %	a
20 %	b

____

15. A die is rolled three times. What is the probability that the outcome of the first roll is an odd number, the outcome of the second roll is an even number and the outcome of the third roll is greater than 3?

____

16. The hockey and football teams of a college play on the same day. The probability that the hockey team wins is $\frac{7}{15}$ and the probability that the football team wins is $\frac{3}{14}$.

What is the probability that both the college's teams lose? ___________

17. The probability that Rafael arrives late at work on a given day is equal to 0.1.
What is the probability that Rafael

a) arrives on time on three consecutive days? ___________

b) arrives late only once in three consecutive days? ___________

18. Consider the following game: "Toss a coin twice and receive in dollars twice the number of tails obtained." A player must pay \$2.50 to play this game.
Is the game favourable to the player?

____

19. Consider the following game: "Toss a coin three times and receive in dollars twice the number of tails obtained". A player pays \$$a$ to play this game. Determine a so that the game is fair.

____

____

20. A coin is tossed until we get either a tails or three consecutive heads. Let X represent the number of tosses. Calculate $E(X)$.

____

21. Consider the following game: "Roll a die once and receive in dollars twice the number shown on the die". A player must pay $7 to play this game.

a) Let X represent the number on the die and Y the player's net gain. Express Y as function of X. ______

b) Determine the probability distribution of the variable Y.

______

c) Is this game fair? ______

22. Consider the following game: "Toss a coin twice and receive in dollars three times the number of tails obtained." A player must pay $2 to play this game. Let Y represent the player's net gain.

a) Determine the probability distribution of the variable Y. ______

b) Explain why this game is not fair. ______

c) How much should a player pay to play this game so that it is fair?

______

23. A student committee organizes a game to raise money for the end of year party. The game consists of rolling a die. If the number rolled is a 6, the player wins $10. If the number is different from 6, the player must pay an amount of money to the committee. What is the minimal amount of money the player should pay the committee so that this game is favourable to the organizing committee?

y		
$P(y)$		

______

______

24. A jar contains 10 yellow marbles, 15 green marbles and 5 black marbles. A game consists of drawing a marble at random and observing the chosen color. If a yellow ball is drawn, the player wins $4, if it is green, he wins $3 and if it is black, he loses $8.

A player pays $1.75 to participate in this game. Is this game fair?

______

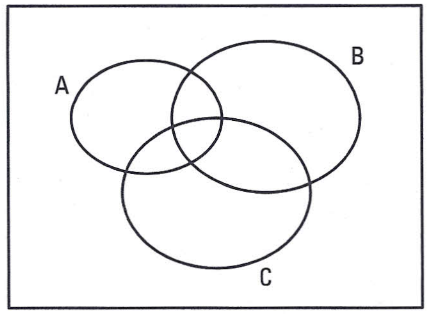
25. In a TV game, a candidate must open a window among 10 windows to discover which prize he will win. There are 4 windows containing a prize of $500, 3 windows containing a prize of $1 000, 2 windows containing a prize of $2 000 and a window containing an unknown prize. A candidate must pay $320 in order to participate in this game. If the candidate raises one of the three windows containing the prize of $1 000, he loses that amount. If not, he wins the indicated prize.

How much should the unknown prize be if this game is fair?

______

5.11 Problems

1. A survey of 1000 people is done regarding magazines A, B and C. The survey shows that 425 people read magazine A; 475 read magazine B; 250 read magazine C; 150 read magazines A and B; 125 read magazines A and C; 75 read magazines B and C; 50 read magazines A, B and C. Calculate the probability that a person in this survey reads



a) two of these magazines. ______

b) none of these magazines. ______

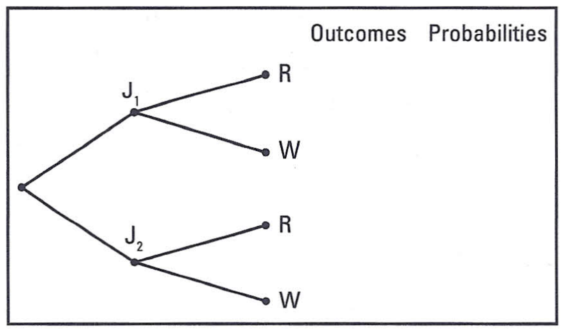
2. A and B are two events such that $P(A) = 0.4$ and $P(A \cup B) = 0.6$. Détermine $P(B)$ if

a) A and B are incompatible. ______

b) A and B are independent. ______

c) event A implies event B. ______

3. Consider two jars. Jar J_1 contains three red balls and two white balls; jar J_2 contains four red balls and one white ball. A jar is chosen at random and a ball is drawn from it.



a) Calculate the probability of drawing a red ball.

______

b) If the ball drawn is red, compute the probability that it came from jar J_1.

______

4. A die is rolled twice. Calculate the following probabilities:

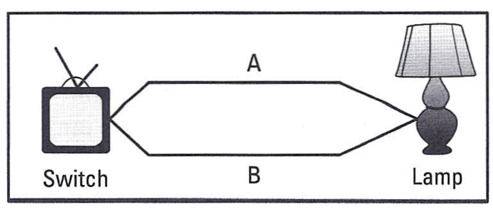
a) "Getting a 6 on the first or second roll." ______

b) "Getting a 6 on the second roll if we got a 6 on the first roll." ______

c) "Getting a sum equal to 7 if we got a 6 on the first roll." ______

d) "Getting a 6 on the first roll if the sum is equal to 7." ______

5. In the electric circuit on the right, cables A and B work independently. Cable A has a 9 out of 10 chance of working, while cable B has an 8 out of 10 chance. When the switch is activated, the lamp does not turn on if both cables are malfunctioning. The switch is activated. What is the probability that the lamp will turn on?



______

6. A die is rolled three times. What is the probability that the outcome of the first roll is an odd number, the outcome of the second roll is an even number and the outcome of the third roll is greater than 3?

____

7. The hockey and football teams of a college play on the same day. The probability that the hockey team wins is $\frac{7}{15}$ and the probability that the football team wins is $\frac{3}{14}$.

What is the probability that both teams lose?

____

8. The probability that Rafael arrives late at work on a given day is equal to 0.1.

What is the probability that Rafael

a) arrives on time on three consecutive days? ____________________

b) arrives late only once in three consecutive days? ____________________

9. A wheel of fortune is divided into 25 isometric sectors. Ten sectors are white, eight are black and the others are red.

A game of chance consists of spinning the wheel. When the wheel stops on a sector that is:

- white, the player receives \$4;
- black, the player receives \$1;
- red, the player loses \$2.

How much must the player bet if this game is fair?

____

10. Two games are offered in a fair. The betting amount for each game is the same.
The first game consists of flipping a coin. The player wins \$6 if the coin lands on tails and loses \$1 if it lands on head.
The second game consists of rolling a die. The player gets \$3 if he receives a divider of 6 and receives a certain amount otherwise.

How much will a player receive, if when rolling a dice, he obtains a result that is not a divider of 6?

____

11. Ten thousand tickets were sold in a lottery. There is 1 prize of \$10 000, 10 prizes of \$1 000 and 100 prizes of \$100. What is the cost of the lottery ticket if the game is fair?

____

12. A jar contains 4 black marbles and 6 red marbles. We successively draw, without replacement, two marbles from the jar. The player receives, in dollars, twice the number of red marbles obtained.

How much must the player bet to participate in this game if the game is fair?

____

Evaluation 5

1. Three candidates Angela, Brittney and Claudia apply for the position of secretary of a union.

Each voting member must write, in order, his preference for the choice of each candidate.

a) 1. How many different ways of voting are there? ____________________

2. Enumerate the different ways.

____

b) The results of the votes have been compiled.

We observe that 20 members chose the order (A, B, C), 12 members chose the order (C, A, B), 10 members chose the order (B, A, C) and 8 members chose the order (A, C, B).

Preference table

c) Determine the winner under

1. a majority ballot. ____________________
2. a plurality ballot. ____________________
3. Borda's method. ____________________
4. Condorcet's criterion. ____________________
5. an elimination ballot. ____________________

2. Using the preference table on the right, determine the winner under

1. a majority ballot. ____________________
2. a plurality ballot. ____________________
3. Borda's method. ____________________

____

4. Condorcet's criterion. ____________________
5. an elimination ballot. ____________________

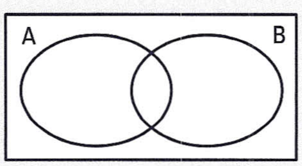
Preference table

Numbers of members	10	14	16	10
1st choice	A	C	B	A
2nd choice	B	A	A	C
3rd choice	C	B	C	B

3. Let A and B be two events associated with a random experiment.

1. Represent the following events using a Venn diagram.
2. Describe each event using the operators $\cap$, $\cup$ or $\overline{\ }$.

a) A and B occur.



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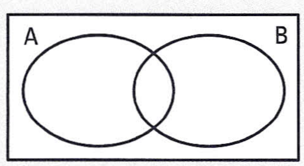
b) A or B occurs.



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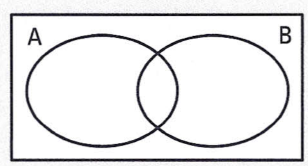
c) B does not occur.



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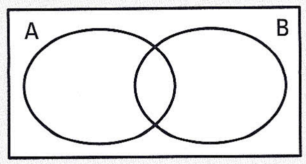
d) A occurs and B does not occur.



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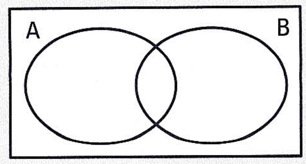
e) A does not occur and B occurs.



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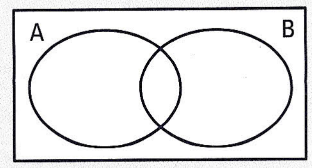
f) A and B do not occur.



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4. A statistical study showed that when a customer enters a convenience store, the probability that she will buy milk is 0.20, that she will buy beer is 0.30 and that she will buy both products is 0.05. A customer enters a convenience store. Calculate the probability that the customer will buy

a) at least one of these two products. ______________

b) only beer. ______________

c) milk if she buys beer. ______________

d) only one of these two products. ______________

5. According to the weather forecast, the probability of precipitations is 60 % for Saturday and 70 % for Sunday. Calculate the probability of the following events.

a) It rains both days. ______________

b) It rains on Saturday only. ______________

c) It rains only one of the two days. ______

d) It doesn't rain during the weekend. __

6. At archery practice, Julian hits the target three out of five times. He shoots three times. Compute the probability of the following events.

a) He misses the target with each shot. _____

b) He hits the target only once. ________

c) He hits the target twice. ____________

d) He hits the target with every shot. _____

7. Every morning, to get to work, an employee has a choice between two means of transportation: bus or car. After many years, he realizes that he takes the bus 4 out of 10 times and the car 6 out of 10 times. Moreover, he observed that when he takes the bus, he has a 2 out of 10 chance of arriving late for work, while he has only a one out of 10 chance of arriving late when he takes the car.

a) What is the probability that the employee arrives late at work? ______________

b) Today, this employee arrived late for work.
What is the probability that he took his car? ______________

8. A card is drawn from a 52-card deck. What are the chances

a) of drawing a red card? ______________

b) of drawing a spade? ______________

c) of drawing an ace? ______________

d) of drawing the ace of spades? ______

9. In each of the following cases, indicate the type of probability (theoretical, empirical, subjective).

a) The probability that it will snow tomorrow is equal to 20%. ______

b) The probability of drawing a heart from a 52-card deck is equal to $\frac{1}{4}$. ______

c) The probability that a student in a school is in secondary 4 is equal to 18%.

______

10. Of the 30 students in a class, 18 students have brown hair, 8 students have green eyes and 3 students have brown hair and green eyes. What is the probability that a student from this class

a) has brown hair but does not have green eyes? ______

b) has green eyes and does not have brown hair? ______

c) does not have green eyes and does not have brown hair? ______

d) has green eyes if he has brown hair? ______

e) does not have green eyes if he does not have brown hair? ______

11. A coin is tossed three times. Let X represent the total number of tails obtained.

a) Determine the distribution of X. ______

b) Calculate $E(X)$. ______

12. A coin is loaded so that $P(\text{tails}) = \frac{1}{3}$. The coin is tossed three times. Let X represent the total number of tails obtained.

a) Determine the distribution of X. ______

b) Calculate $E(X)$. ______

13. A die is rolled and you receive in dollars the number showing on the die. How much should you pay to play this game if it is favourable to the player?

______

14. Two dice are rolled and you receive in dollars the sum of the numbers showing on the dice. Is this game favourable to a player who pays $7.50 to play the game?

______

15. Two dice are rolled. If the sum of the numbers is even, you win in dollars an amount equal to this sum; if the sum is odd, you lose in dollars an amount equal to this sum. Analyze this game.

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