

5.2 Voting procedures

ACTIVITY 1 Voting procedures

The 40 members of the board of directors of a company must vote to elect the president of the board. Three candidates Ann, Bernard and Caroline apply for the position. Each voting member must write, in order, his preference for the choice of each candidate.

Votes	16	14	10
1st choice	A	C	B
2nd choice	B	A	A
3rd choice	C	B	C

The results are compiled in the table on the right. Thus, we observe that 16 members chose the order (A, B, C), 14 chose the order (C, A, B), 10 chose the order (B, A, C) and none chose the order (A, C, B) or (C, B, A) or (B, C, A).

- a) In a **majority ballot**, the winner is the one who gets more than half the votes.
1. Is there a candidate who wins in a majority ballot? _____
 2. On how many ballots, at the minimum, must a candidate be rated as 1st choice to win a majority ballot? _____

- b) In a **plurality ballot**, the winner is the one who gets the most votes, in other words the one who has been rated as 1st choice most often.
- Which of the three candidates has been rated as 1st choice most often and would win a plurality ballot? _____

- c) **Borda's method** is a procedure in which points are allocated to each candidate. If, for example, 2 points are allocated to the preferred candidate, 1 point to the following one and 0 to the last one, the winner is the candidate who earns the most points.

Determine for each candidate the total number of points and then deduce the winner according to Borda's method.

- d) Under **Condorcet's criterion**, the winner is the one who wins over the other candidates in a head-to-head confrontation.

1. Determine which candidate would be the winner in a confrontation between
 - 1) Ann and Bernard. _____
 - 2) Ann and Caroline. _____
 - 3) Bernard and Caroline. _____
2. Which candidate would therefore be the winner under Condorcet's criterion? _____

- e) In an **elimination ballot**, the winner is determined by the following procedure:
1. In the 1st step, first place votes are counted for each candidate and the one who holds the least number of votes is eliminated.

Determine the 1st place votes for each candidate and verify that Bernard is eliminated.

2. The 2nd step consists in eliminating, from the preference table, the candidate that was eliminated in the 1st step and to allocate the first place votes of the eliminated candidate to the candidate that follows him, and then to recount the first place votes.

Bernard being eliminated, Ann, who follows him in the choice (B, A, C) will reap Bernard's 10 first place votes. Ann therefore gets a total of 26 first place votes (16 + 10), which represents a majority of votes. Who is therefore the winner under an elimination vote?

VOTING PROCEDURES

Let us illustrate the different voting procedures with the following example:

Three villages A, B and C in a region are candidates to determine the village where a school will be built. The results of a poll are represented in the preference table on the right.

45 %	35 %	20 %
A	B	C
B	A	B
C	C	A

There exists different voting procedures:

- **Majority ballot:**
The winning candidate is the one who received more than half the votes.
Ex.: None of the villages obtains a majority. We do not get a winning village under a majority ballot.
- **Plurality ballot:**
The winning candidate is the one who received the greatest number of votes.
Ex.: Village A who received the greatest number of votes is the winner under a plurality ballot.
- **Borda's method:**
Weighted voting system. Each voter constructs a list of n candidates in order of preference. For example, if there are 4 candidates, we allocate 3 points to the first one, 2 points to the second one, etc... and 0 points to the last one. The candidate(s), whose score is the greatest, wins the election.
Ex.: Let us allocate 2 points to the village in the 1st place, 1 point to the 2nd place and 0 point to the 3rd place. A receives 125 points ($45 \times 2 + 35 \times 1 + 20 \times 0$), B receives 135 points ($45 \times 1 + 35 \times 2 + 20 \times 1$), C receives 40 points ($45 \times 0 + 35 \times 0 + 20 \times 2$). Village B is the winner under Borda's method.
- **Condorcet's criterion:**
Voting system in which the sole winner is the one, if he exists, who, when compared to each other candidate in turn, proves to be the preferred candidate every time.
Ex.: If villages B and A are confronted, B wins with 55%. If B and C are confronted, B wins with 80%. Village B is the preferred candidate every time and is declared the winner under Condorcet's criterion.
- **Elimination ballot:**
In the 1st step,
– we count for each candidate the number of 1st place votes,
– we reject the one who holds the fewest votes.

In the 2nd step,

- we eliminate the rejected candidate from the preference table,
- we allocate the number of 1st place votes of the rejected candidate to the following candidate,
- we recount the number of 1st place votes.

If a candidate gets a majority of votes, he wins the election. Otherwise, we eliminate the one who has the fewest votes and we repeat the procedure until we obtain a winner.

Ex.: We reject candidate C since it obtained only 20% of the 1st place votes.

Candidate B which follows then receives an additional 20% of the 1st place votes and thus obtains 55% of the 1st place votes in the new preference table.

Village B is declared the winner under an elimination vote.

45 %	55 %
A	B
B	A

1. The 20 members of the board of directors of a company must vote to elect the president of the board. Three candidates Andy, Bridget and Clara apply for the position. Each voting board member must write, in order of preference, his choice of candidate.

- a) How many distinct results in total can be observed in this vote?

The 20 results have been compiled in the following preference table:

Votes	4	3	2	3	5	3
1st choice	A	A	B	B	C	C
2nd choice	B	C	A	C	A	B
3rd choice	C	B	C	A	B	A

- b) Determine the winner, and justify your answer, under

1. a majority ballot. _____
2. a plurality ballot. _____
3. Borda's method. _____
4. Condorcet's criterion. _____
5. an elimination ballot. _____

2. Three candidates a , b and c , apply for the position of president of a company. The 10 members of the board of directors have ranked the candidates in order of their preference.

The table on the right gives the results. Is the elected candidate the same whether a majority ballot or Borda's method is used?

Votes	6	3	1
1st choice	a	b	c
2nd choice	b	c	b
3rd choice	c	a	a

3. Three candidates a , b and c apply for the position of secretary of a union. The results are given in the preference table on the right. Is the elected candidate the same whether Condorcet's criterion or a plurality ballot is used?

Votes	33 %	29 %	26 %	12 %
1st choice	a	b	c	c
2nd choice	c	c	b	a
3rd choice	b	a	a	b

4. Three candidates a , b and c are running to become class president. The results are given in the preference table on the right. Is the elected candidate the same whether Condorcet's criterion or Borda's method is used?

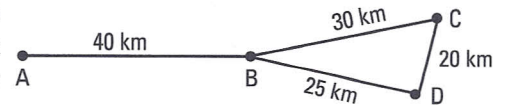
Votes	16	8	4
1st choice	a	b	c
2nd choice	b	c	b
3rd choice	c	a	a

5. The 40 members of the board of directors of a company must vote to elect the president of the board. Four candidates Ann, Bob, Cindy and David apply for the position. Each voting member must write his order of preference for each candidate.
- a) How many distinct results in total can be observed in this vote?

The 40 results have been compiled in the following preference table:

Votes	18	16	4	2
1st choice	A	B	C	D
2nd choice	B	C	B	B
3rd choice	C	D	D	C
4th choice	D	A	A	A

- b) Determine the winner under
- a majority ballot. _____
 - a plurality ballot. _____
 - Borda's method. _____
 - Condorcet's criterion. _____
 - an elimination ballot. _____
6. We want to determine in which one of the four villages on the right a medical centre will be built. Each resident wishes that the centre be closest to his village. A survey gave the preference table on the right.



Determine the winning village under

- a majority ballot. _____
- a plurality ballot. _____
- Borda's method. _____
- Condorcet's criterion. _____
- an elimination ballot. _____

Village A (40 %)	Village B (35 %)	Village C (15 %)	Village D (10 %)
A	B	C	D
B	D	D	C
D	C	B	B
C	A	A	A

7. For each of the following preference tables, determine the winner of the vote according to the voting procedure.

a)

Votes	6	7	7
1st choice	A	B	C
2nd choice	B	C	A
3rd choice	C	A	B

1. a majority ballot. _____
2. a plurality ballot. _____
3. Borda's method. _____
4. Condorcet's criterion. _____
5. an elimination ballot. _____

b)

Votes	9	3	8
1st choice	A	B	C
2nd choice	B	C	A
3rd choice	C	A	B

1. a majority ballot. _____
2. a plurality ballot. _____
3. Borda's method. _____
4. Condorcet's criterion. _____
5. an elimination ballot. _____

c)

Votes	7	11	2
1st choice	A	B	C
2nd choice	B	C	A
3rd choice	C	A	B

1. a majority ballot. _____
2. a plurality ballot. _____
3. Borda's method. _____
4. Condorcet's criterion. _____
5. an elimination ballot. _____

d)

Votes	9	5	6
1st choice	A	B	C
2nd choice	B	C	A
3rd choice	C	A	B

1. a majority ballot. _____
2. a plurality ballot. _____
3. Borda's method. _____
4. Condorcet's criterion. _____
5. an elimination ballot. _____

e)

Votes	9	7	4
1st choice	A	B	C
2nd choice	B	C	A
3rd choice	C	A	B

1. a majority ballot. _____
2. a plurality ballot. _____
3. Borda's method. _____
4. Condorcet's criterion. _____
5. an elimination ballot. _____

f)

Votes	7	7	6
1st choice	A	B	C
2nd choice	B	C	A
3rd choice	C	A	B

1. a majority ballot. _____
2. a plurality ballot. _____
3. Borda's method. _____
4. Condorcet's criterion. _____
5. an elimination ballot. _____

g)

Votes	8	7	5
1st choice	A	C	B
2nd choice	B	B	C
3rd choice	C	A	A

1. a majority ballot. _____
2. a plurality ballot. _____
3. Borda's method. _____
4. Condorcet's criterion. _____
5. an elimination ballot. _____

h)

Votes	9	8	3
1st choice	A	C	B
2nd choice	B	B	C
3rd choice	C	A	A

1. a majority ballot. _____
2. a plurality ballot. _____
3. Borda's method. _____
4. Condorcet's criterion. _____
5. an elimination ballot. _____

i)

Votes	9	6	5
1st choice	A	B	C
2nd choice	B	C	B
3rd choice	C	A	A

1. a majority ballot. _____
2. a plurality ballot. _____
3. Borda's method. _____
4. Condorcet's criterion. _____
5. an elimination ballot. _____