1 For each case, determine the winner based on:
a) the Borda count

Results of the ranking of three activities

| Number of people who <br> ranked the activities in this way | 17 | 15 | 12 | 8 |
| :---: | :---: | :---: | :---: | :---: |
| 1st choice | Reading | Walking | Walking | Movies |
| 2nd choice | Walking | Reading | Movies | Reading |
| 3rd choice | Movies | Movies | Reading | Walking |

b) the Condorcet method

Results of the ranking of three sports

| Number of people who <br> ranked the sports in this way | 20 | 14 | 11 | 9 |
| :---: | :---: | :---: | :---: | :---: |
| 1st choice | Soccer | Soccer | Hockey | Tennis |
| 2nd choice | Tennis | Hockey | Tennis | Soccer |
| 3rd choice | Hockey | Tennis | Soccer | Hockey |

c) the elimination method

Results of the ranking of three desserts

| Number of people who <br> ranked the desserts in this way | 17 | 15 | 12 | 10 | 8 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1st choice | Fruit | Jello | Jello | Yogurt | Yogurt | Fruit |
| 2nd choice | Yogurt | Fruit | Fruit | Fruit | Jello | Yogurt |
| 3rd choice | Jello | Yogurt | Yogurt | Jello | Fruit | Jello |

2 A school's administrators must choose an end-of-year field trip from among four possibilities. To do so, they conduct a student survey. The following are the results:

Results of the ranking of four field trips

| Number of people <br> who ranked <br> the field trips <br> in this way | 27 |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1st choice | Show | Outdoor activity | Movie | Museum | Museum | Outdoor activity |
| 2nd choice | Movie | Movie | Outdoor activity | Show | Movie | Museum |
| 3rd choice | Museum | Show | Show | Outdoor activity | Show | Movie |
| 4th choice | Outdoor activity | Museum | Museum | Movie | Outdoor activity | Show |

Which field trip is chosen if the winner is determined by using:
a) the Condorcet method?
b) the Borda count?

3 The following diagram illustrates the roads linking four villages. The government wants to build a school in one of the villages that will serve all four villages.


In partnership with other organizations, the United Nations Children's Fund, better known as UNICEF, made universal primary and secondary education one of the Millennium Development Goals.
a) Complete the adjacent table considering that all the inhabitants of a given village would prefer that the school be located as close as possible to their village.
b) Which village is chosen using:

1) plurality voting?
2) the Borda count?

Choice of school location

| Number of people <br> Who ranked the vilages <br> in this way | 4000 |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 1st choice | A |  |  |  |
| 2nd choice | D |  |  |  |
| 3rd choice | C |  |  |  |
| 4th choice | B |  |  |  |

3) the Condorcet method?
c) The government plans to build a $13-\mathrm{km}$ road between Villages $\mathbf{A}$ and $\mathbf{B}$ as well as a hospital serving the four villages. Given that all the inhabitants of a village would prefer that the hospital be located as close as possible to their home, determine which village will be chosen using:
4) the majority rule
5) plurality voting
6) the Borda count

4 A group of friends wants to predict, which among four hockey teams, will perform the best. To do so, each person ranks the teams in order of preference. The following are the results:

Results of the ranking of the four teams

| Number of friends who ranked <br> the teams in this way | 9 | 7 | 7 | 6 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1st choice | Bruins | Penguins | Bruins | Hurricanes | Canadiens |
| 2nd choice | Hurricanes | Canadiens | Canadiens | Canadiens | Penguins |
| 3rd choice | Penguins | Hurricanes | Hurricanes | Penguins | Hurricanes |
| 4th choice | Canadiens | Bruins | Penguins | Bruins | Bruins |

Which team will be chosen based on:
a) the majority rule?
b) plurality voting?
c) the Borda count?

5 Elections are held in a country whose parliament contains 10 seats. This country is divided into 10 districts that each contains the same number of voters. The following are the results of the election:

Election results

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | 10\% | 35\% | 23\% | 24\% | 14\% | 7\% | 35\% | 40\% | 33\% | 33\% |
| B | 43\% | 33\% | 31\% | 51\% | 11\% | 48\% | 12\% | 28\% | 31\% | 33\% |
| C | 47\% | 32\% | 46\% | 25\% | 75\% | 45\% | 53\% | 32\% | 36\% | 34\% |

a) 1) Determine the composition of this country's parliament if one seat is attributed to each district according to plurality voting.
2) Does the party in power have a majority? Explain your answer.
b) 1) Determine the composition of this country's parliament if seats are attributed according to proportional representation.
2) Does the party in power have a majority? Explain your answer.

6 In order for an organization to complete its board of directors, its 400 members must elect two people out of four candidates. The following are the results of the vote:

Voting results

| Number of members who <br> ranked the candidates <br> in this way | 120 | 108 | 98 | 50 | 24 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1st choice | Angelo R. | Jeanne C. | Julie P. | Marcel G. | Julie P. |
| 2nd choice | Julie P. | Marcel G. | Marcel G. | Angelo R. | Jeanne C. |
| 3rd choice | Jeanne C. | Julie P. | Angelo R. | Jeanne C. | Angelo R. |
| 4th choice | Marcel G. | Angelo R. | Jeanne C. | Julie P. | Marcel G. |

The elimination method is used to determine the two new members of the board of directors. Who are they?

The European Union (EU) brings together some thirty European countries into an economic and political partnership. The Council of the European Union is made up of members from the national governments of all these countries. The number of votes to which each country is entitled within the Council is based on the size of the country's population.


7 PROVINCIAL ELECTIONS During Quebec provincial elections, the population of each of the 125 districts elects a member according to plurality voting. The table below lists the results of the 2007 and 2008 elections.

Provincial election in Québec

| Party | 2007 election |  | 2008 election |  |
| :--- | :---: | :---: | :---: | :---: |
|  | Number <br> of seats | Number <br> of votes | Number <br> of seats | Number <br> of votes |
| Liberal Party | 48 | 1313664 | 66 | 1366046 |
| Parti québécois | 36 | 1125546 | 51 | 1141751 |
| Action démocratique du Québec | 41 | 1224412 | 7 | 531358 |
| Québec solidaire | 0 | 144418 | 1 | 122618 |
| Québec Green Party | 0 | 152885 | 0 | 70393 |
| Other parties | 0 | 9693 | 0 | 14167 |

a) Which party led the government in:

1) 2007 ?
2) 2008 ?
b) Did this party have a majority in:
3) 2007? Explain your answer.
4) 2008? Explain your answer.
c) Describe the composition of Québec's parliament had the members been elected based on proportional representation:

The Parliament of Québec, also called the National Assembly, contains 125 members elected in the 125 electoral districts. The laws of Québec are voted on in the National Assembly.

1) in 2007
2) in 2008

8 A municipality's 10 councillors are elected by proportional representation. To do so, each voter chooses 10 candidates out of 30 . The table below represents a summary of the results obtained.
a) How many seats were won by:

1) Party $A$ ?
2) Party B ?
3) Party $C$ ?

The seats won by a party are attributed to the candidates who received the most votes for their party.
b) Which candidates will have seats for the following parties:

1) Party $A$ ?
2) Party B?
3) Party C ?

| Party A |  | Party B |  | Party C |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Candidate | Number <br> of votes <br> received | Candidate | Number <br> of votes <br> received | Candidate | Number <br> of votes <br> received |
| 1 | 4032 | 1 | 6356 | 1 | 345 |
| 2 | 1250 | 2 | 953 | 2 | 7684 |
| 3 | 120 | 3 | 1436 | 3 | 546 |
| 4 | 5504 | 4 | 543 | 4 | 2543 |
| 5 | 240 | 5 | 326 | 5 | 746 |
| 6 | 1302 | 6 | 865 | 6 | 1376 |
| 7 | 2074 | 7 | 2437 | 7 | 3276 |
| 8 | 5244 | 8 | 432 | 8 | 4527 |
| 9 | 1030 | 9 | 3249 | 9 | 1438 |
| 10 | 270 | 10 | 1321 | 10 | 645 |

9 DUNCAN BLACK Scottish economist Duncan Black applied the Borda count when the Condorcet method did not produce a winner. For each of the following cases, determine the winner using this reasoning.
a)
Election of a representative for a group of swimmers

| Number of voters who <br> ranked the candidates <br> in this way | 28 | 20 | 16 | 12 |
| :---: | :---: | :---: | :---: | :---: |
| 1st choice | C | B | D | D |
| 2nd choice | B | C | A | B |
| 3rd choice | A | D | C | C |
| 4th choice | D | A | B | A |

b)
Election of a representative for an organization

| Number of voters who <br> ranked the candidates <br> in this way | 23 | 17 | 10 | 8 | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1st choice | A | B | C | C | B |
| 2nd choice | B | C | A | B | A |
| 3rd choice | C | A | B | A | C |

10 ALABAMA PARADOX In some countries, the number of parliamentary seats allotted for each district is proportional to that district's population.

The following is some information regarding four districts in a country with a 162-seat parliament.

Attribution of seats

| District | Population | Minimum number of seats allotted | Remainder |
| :---: | :---: | :---: | :---: |
| A | 60000 | $162 \times \frac{60000}{100000}=97.2$ or 97 seats | 0.2 |
| B | 30000 |  |  |
| C | 9000 |  |  |
| D | 1000 |  |  |
| Total | 100000 |  |  |

a) Complete the table above.
b) Indicate the number of seats attributed to each of the four districts.
c) If the number of parliamentary seats were increased to 163 , show that the number of seats allotted to District D would decrease.

In performing simulations of proportional representation in the United States, it was observed that the State of Alabama would receive 8 seats in a 299-seat parliament and 7 seats in a 300-seat parliament. This situation gave rise to the Alabama paradox, which demonstrates that in some cases, an increase in the number of parliamentary seats could result in a district being allotted fewer seats than it previously possessed.


3 The following is a way to calculate the degree of satisfaction of an electorate after a vote:

When the elected candidate is defined as follows, note the following:

- A voter's 1st choice, the voter is very satisfied. Each very satisfied voter is worth 3 points.
- A voter's 2 nd choice, the voter is satisfied. Each satisfied voter is worth 2 points.
- A voter's 3rd choice, the voter is dissatisfied. Each dissatisfied voter is worth 1 point.

The total number of electorate points is determined and then divided by the number of voters. The result corresponds to the electorate's degree of satisfaction.

The table below presents the results of an election.

Election results

| Number of voters <br> who ranked the candidates <br> in this way | 50 | 30 | 27 | 24 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1st choice | C | B | A | A | B |
| 2nd choice | B | C | B | C | A |
| 3rd choice | A | A | C | B | C |

a) Who wins according to:

1) plurality voting?
2) the Borda count?
3) the elimination method?
b) For each voting procedure listed in a), determine the number of voters who are:
4) very satisfied
5) satisfied
6) dissatisfied
c) Of the voting procedures listed in a), which, in this situation, generates:
7) the highest degree of electorate satisfaction?
8) the lowest degree of electorate satisfaction?

In certain countries, elections take place in the presence of international observers because the integrity of the votes
 could be at risk.

4 A country's parliament is made up of one seat for each district. After an election, the winning party obtains almost all of the seats in the parliament. However, this party received the fewest votes.
a) Which voting procedure was used?
b) Explain how this situation could come about.

5 CLYDE COOMBS Around 1954, mathematician Clyde Coombs developed a variation of the elimination method that consists of eliminating the candidate who received the most last-place votes and attributing those votes to the candidates that follow. This procedure is repeated until a candidate has over half the votes. Below are the results of an election:

Election results

| Number of voters who <br> ranked the candidates <br> in this way | 84 | 52 | 34 | 30 |
| :---: | :---: | :---: | :---: | :---: |
| 1st choice | A | B | D | C |
| 2nd choice | B | C | C | D |
| 3rd choice | C | D | B | B |
| 4th choice | D | A | A | A |

American Clyde Coombs (19121988) founded the Mathematical Psychology program at the University of Michigan.
a) Determine the winner using the Coombs method.
b) Verify that the same winner would not be obtained by using the elimination method.

6 The table below presents the votes received in an election based on preferences.

| Voting results |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of voters who <br> ranked the candidates <br> in this way | 100 | 60 | 57 | 48 | 20 |  |
| 1st choice | A | B | C | C | B |  |
| 2rd choice | B | A | B | A | C |  |
| 3rd choice | C | C | A | B | A |  |

a) Who is the winner of this election according to:

1) plurality voting?
2) the Borda count?
3) the Condorcet method?
b) Explain why it is important to determine the voting procedure before an election.

7 The following table shows the results of a survey taken before an election wherein the winning party is determined by means of the elimination method.

Survey results

| Group |  | 1 | 2 | 3 | 4 |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: |
| Group preferences | 1st choice | Party A | Party C | Party B | Party B |
|  | 2nd choice | Party B | Party A | Party C | Party A |
|  | 3rd choice | Party C | Party B | Party A | Party C |
| Percentage of voters in <br> this group |  | 35 | 31 | 24 | 10 |

Following this survey, Party A intensified its campaign and managed to convince half the voters in Group 4. They now have the same preferences as Group 1.
Show that Party A would have been better off not intensifying its campaign.

8 INDEPENDENCE OF IRRELEVANT ALTERNATIVES The criterion of independence of irrelevant alternatives stipulates that if an election determines a winning candidate, the winner must remain the same even if one of the candidates withdraws his or her candidacy. The result of an election are as follows:

Election results

| Number of voters who ranked <br> the canddates in this wey | 52 | 41 | 35 | 33 | 29 | 21 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Ist choice | B | A | A | D | B | C |
| 2nd choice | A | B | D | B | A | A |
| 3rd choice | D | C | C | C | C | D |
| 4th choice | C | D | B | A | D | B |

a) Who is the winner of this election according to:

1) the Borda count?
2) the Condorcet method?
3) the elimination method?
b) Candidate D withdraws his candidacy. Who is now the winner according to:
4) the Borda count?
5) the Condorcet method?
6) the elimination method?
c) In this example, which of the voting procedures does not respect the criterion of independence of irrelevant alternatives?

9 Four candidates run in a country's presidential election. The following are the results of this election in which the winner is determined by the elimination method.

A country's presidential election

| Percentrge of voters who <br> ranked the candidates <br> in this way | 30 | 22 | 21 | 10 | 9 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1st choice | A | B | A | D | C | B |
| 2nd choice | B | C | B | B | B | D |
| 3rd choice | D | D | C | C | D | C |
| 4th choice | C | A | D | A | A | A |

a) Explain how this situation could generate political tension within the electorate.
b) Explain why the election of Candidate $\mathbf{B}$ would have been a more satisfactory result for the electorate.


Dubbed the "Iron Lady," Margaret Thatcher was Prime Minster of the United Kingdom from 1979 to 1990.
c) Which method would allow Candidate $\mathbf{B}$ to be elected?

Each country has its own political system. While the United States is led by a president and Canada is led by a prime minister, some countries, like France and Russia, have both a president and a prime minister.

10 The following are the results of a municipal election that took place in a city's two districts.
Results of a municipal election

|  |  |  | District 1 |  |  | District 2 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of voters who ordered <br> the candidates in this way | 4800 | 4200 | 3800 | 2200 | 1200 | 1100 |  |  |  |
| 1st choice | C | A | D | D | A | B |  |  |  |
| 2nd choice | D | C | B | C | C | C |  |  |  |
| 3rd choice | A | B | A | A | B | A |  |  |  |
| 4th choice | B | D | C | B | D | D |  |  |  |

In each district, the winner is determined by the elimination method.
a) Who is the winner:

1) in District 1?
2) in District 2?
b) Had the region not been divided into districts, would the result of the vote have been different? Explain your answer.
c) Does this phenomenon occur when the winner is determined using the Borda count?

6 On average, a basketball player makes 75\% of her free throws.
a) If she shoots 4 free throws during a game, determine the probability of the following:

1) She will make all 4 free throws.
2) She will make the last 2 free throws given that she made the first two.
3) She will not make the first 2 throws.
4) She will make the first 3 throws.
b) Does making the first free throw have an effect on making the second? Explain your answer.

In basketball, a free throw is a penalty throw that is granted to a victim of prohibited contact from an opposing player when attempting to complete a basket. A free throw grants two throws that are taken 4.6 m from the basket.

7 The 50 owners of a co-op property must decide on a new colour for the building's exterior. The following table presents a summary of the votes cast according to the owners' preferences.

Voting results

| Number of owners who ranked <br> the colours in this way | 17 | 15 | 13 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1st choice | White | Grey | Blue | White |
| 2nd choice | Blue | Blue | Grey | Grey |
| 3rd choice | Grey | White | White | Blue |

Which colour will the exterior be painted if it is determined using:
a) the Borda count?
b) the Condorcet method?
c) the elimination method?

Typical houses in the Plateau
Mont-Royal, in Montréal.


8 In a wallet, there are 3 quarters, 4 loonies and 2 toonies. 2 coins are drawn consecutively from the wallet without replacement.
a) Create a probability tree illustrating this situation.
b) What is the probability that the total value of the coins selected is \$4?
c) What is the probability of drawing a loonie on the second selection given that a loonie was picked on the first selection?

The caribou has adorned the quarter since 1936. Prior to that, the coin featured two crossed maple branches, a design that also appeared on the dime and the 50 -cent coin.

9 To elect the head of an NGO's board of directors, the members can propose the person or people whom they would accept in the position. The person whose name appears the most often is then chosen. The table below presents the results obtained.

Election results

| Number of members who proposed this list of candidates | 39 | 32 | 26 | 18 | 12 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| List of candidates | Gregory | Jean | Fatima | Charles | Marcel |
|  | Audrey | Claude | Claude |  | Genevieve |
|  | Genevieve | Genevieve |  |  | Brittany |
|  | Anne | Marcel |  |  |  |

a) What voting procedure is used by this organization?
b) Who will lead the board of directors?


Amnesty International, Human Rights Watch and Action Against Hunger are all examples of NGOs (non-governmental organizations) that act on a global scale in the areas of human rights and the fight against hunger.

10 A country's parliament is divided into 10 districts and comprises 20 seats which are allocated as follows.

- 10 seats are allocated to the political parties according to the number of votes received.
- 1 seat is allocated for each district according to plurality voting.

The following are the results of this country's last election:
Number of votes received by each party in each district

|  | District | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Party Total |  |  |  |  |  |  |  |  |  |  |  |
| A | 2500 | 1436 | 2134 | 2546 | 4365 | 873 | 3254 | 2456 | 342 | 3084 | 22990 |
| B | 2453 | 2765 | 1434 | 2987 | 642 | 1664 | 2876 | 2543 | 1987 | 3879 | 26230 |
| C | 3215 | 2322 | 3654 | 132 | 1007 | 2997 | 2764 | 2767 | 4007 | 3900 | 26765 |

What is the composition of this parliament?

The following is a table of the preferences expressed during a vote:
Voting results

| Number of voters who ranked <br> the candidates in this way | 74 | 64 | 50 | 42 |
| :---: | :---: | :---: | :---: | :---: |
| 1st choice | C | A | B | D |
| 2nd choice | D | B | C | C |
| 3rd choice | B | C | D | A |
| 4th choice | A | D | A | B |

One person is chosen at random from those who voted. What is the probability of choosing a person:
a) whose 1 st choice corresponds to the winner of the election according to the plurality voting?
b) whose 1 st or 2 nd choice corresponds to the winner of the election according to the Borda count?


In Québec, the Chief Electoral Officer is responsible for the conduct of elections and referendums. This person ensures that the rules of political financing are respected and guarantees that electoral laws are fully enforced. This person is the arbitrator of Québec's electoral system.
12. Ali and Janice are communicating using walkie-talkies that have a range of 500 m . Ali is at the intersection of Garnier Street and Joliette Street, and Janice places herself at a random location in the area represented by the rectangular grid below. The side of each square measures 100 m .

a) What is the probability that Janice is located:

1) in the Brulotte District given that Ali and Janice cannot communicate?
2) in the Brulotte District given that Ali and Janice can communicate?
3) in the Beaulac District given that Ali and Janice cannot communicate and that Janice is not in the Hamel District?
b) What is the probability that Ali and Janice:
4) can communicate given that Janice is located in the Fournier District?
5) cannot communicate given that Janice is not located in the Jolicoeur District or in the Hamel District?

13 The following is some information about high school graduation at two local high schools in a city:

| School © (A) students |  |  |  |
| :--- | :---: | :---: | :---: |
|  | Number <br> of <br> girls | Number <br> of <br> boys | Total |
| Received a diploma | 550 | 1250 | 1800 |
| Did not receive a diploma | 50 | 250 | 300 |
| Total | 600 | 1500 | 2100 |

School (B) students

|  | Number <br> of <br> girls | Number <br> of <br> boys | Total |
| :--- | :---: | :---: | :---: |
| Received a diploma | 630 | 150 | 780 |
| Did not receive a diploma | 450 | 150 | 600 |
| Total | 1080 | 300 | 1380 |



Towards the end of the 13th century and at the beginning of the 14th century, the biretta, the predecessor of the traditional graduation cap, was commonly worn by lawyers, judges and doctors.

This information is used to determine the probability that a student obtained a high school diploma.
a) What is the probability that a student in this city obtained a high school diploma:

1) given that he is a boy?
2) given that she is a girl?
b) Between a boy and a girl in this city, who has a higher probability of obtaining a diploma?
c) What is the probability that a student in this city obtained a high school diploma given that:
3) she is a girl from School (A)?
4) he is a boy from School (A)?
5) she is a girl from School (B)?
6) he is a boy from School (B)?
d) Between a boy and a girl who studied:
7) at School (A), who has a higher probability of obtaining a diploma?
8) at School (B), who has a higher probability of obtaining a diploma?
e) Explain why the answers to questions b) and d) seem to contradict each other.

14 The Condorcet method can be illustrated using a directed graph in which the following is true:

- Each vertex corresponds to a candidate.
- Each edge corresponds to a preference.

For example, an edge with a value of 5 directed from $A$ to $B$ means that 5 voters prefer $A$ to $B$.
a) Complete the graph below, which represents the results of an election.


Election results

| Number of voters who ranked <br> the candidates in this way | 62 | 45 | 23 | 21 |
| :---: | :---: | :---: | :---: | :---: |
| 1st choice | A | B | D | C |
| 2nd choice | B | C | C | D |
| 3rd choice | C | D | B | B |
| 4th choice | D | A | A | A |

Based on this graph, a duel graph can be created in which only the highest-value edges between two vertices are retained.
b) Complete the adjacent duel graph.
c) Graphically, how can the winner of this election be identified?


The table and graph below present a summary of the results of another election.
Election results

| Number of voters who ranked <br> the candidates in this way | 15 | 13 | 10 | 9 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1st choice | B | D | C | A | B |
| 2nd choice | A | B | D | C | C |
| 3rd choice | D | C | A | B | A |
| 4th choice | C | A | B | D | D |


d) Using the results above, complete the adjacent duel graph in which each edge represents the number of votes by which a candidate won the duel with another candidate. For example, an edge directed from $A$ to $B$ of value 5 means that $A$ won the duel over B by 5 votes.
e) Explain why it is impossible to determine the winner using
 the Condorcet method.
f) One way to determine the winner consists of eliminating the edges of lowest value, one by one, until a candidate who does not lose any duel has been identified.
Determine the winner by applying this method.

15 A district's voters must elect two representatives from among four candidates. The following are the results:

| Election results |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Number of voters who <br> ranked the candidates <br> in this way | 5478 | 4327 | 3480 | 2530 | 1657 |
| 1st choice | Julie P. | Pauline F. | Antoine R. | Julie P. | Samir H. |
| 2nd choice | Antoine R. | Antoine R. | Pauline F. | Pauline F. | Antoine R. |
| 3rd choice | Samir H. | Samir H. | Julie P. | Antoine R. | Pauline F. |
| 4th choice | Pauline F. | Julie P. | Samir H. | Samir H. | Julie P |

Determine the winner if the following occurs:
a) The Borda count is used, and the two candidates with the most points are chosen.
b) The elimination method is used, and the two remaining candidates are chosen.

16 A country's 150-seat parliament is divided into 10 districts. The table below provides information on this topic.

Distribution of the population by district

| District | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Percentage of <br> voters living in <br> this district | 14 | 8 | 10 | 10 | 12 | 8 | 6 | 12 | 12 | 8 |

The following are the results of an election that took place in this country:

Election results

| District | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | 10\% | 35\% | 23\% | 24\% | 14\% | 7\% | 35\% | 40\% | 33\% | 33\% |
| B | 43\% | 33\% | 31\% | 51\% | 11\% | 48\% | 12\% | 28\% | 31\% | 33\% |
| C | 47\% | 32\% | 46\% | 25\% | 75\% | 45\% | 53\% | 32\% | 36\% | 34\% |

a) Determine the percentage of votes received by each party in this country.
b) 1) Determine the composition of the parliament if proportional representation is used.
2) Will the party that wins the most seats have a majority? Explain your answer.
c) The number of seats allotted to each district is proportional to its population, and each district's seats are attributed to the parties based on proportional representation.

1) Determine the composition of the parliament.
2) Will the party that wins the most seats have a majority? Explain your answer.

5 In analyzing the results of a vote, the majority rule stipulates that if a candidate is preferred by over half of the voters, this candidate must be declared the winner.

Show, using three candidates, that the Condorcet method respects this rule.

6 The tables below show the results obtained after 1200 tosses of a balanced die and a loaded die.

| Balanced die |  |
| :---: | :---: |
| Result | Frequency |
| 1 | 200 |
| 2 | 200 |
| 3 | 200 |
| 4 | 200 |
| 5 | 200 |
| 6 | 200 |


| Loaded die |  |
| :---: | :---: |
| Result | Frequeney |
| 1 | 50 |
| 2 | 250 |
| 3 | 100 |
| 4 | 50 |
| 5 | 150 |
| 6 | 600 |



Peter chooses a die at random and tosses it twice. What is the probability that he chose the loaded die given that he rolled 6 both times?

7 When voters deem that their preferred candidate has no chance of winning, they may proceed with a strategic vote.

Strategic voting consists of voting for a party that is liked less in order to lower the chances that a party that is liked even less will win the election.

A newspaper publishes the results of a survey prior to an election wherein the winning party will be determined by the elimination method. The following are the results:

Survey results

| Group |  | 1 | 2 | 3 |
| :--- | :---: | :---: | :---: | :---: |
| Group preferences | 1st choice | Party B | Party C | Party A |
|  | 2nd choice | Party C | Party A | Party D |
|  | 3rd choice | Party A | Party D | Party B |
|  | 4th choice | Party D | Party B | Party C |
| Percentage of voters in this group | 47.6 | 28.6 | 23.8 |  |

Following this survey, the voters in Group 2 deem that their candidate will not win the election and decide to vote with the voters in Group 3.
Submit a short article to this newspaper explaining why the elimination method encourages strategic voting in this situation.

8 In 1994, the European Parliament was composed of 567 seats divided among the following 12 countries:

Population in Europe in 1994

| Country | Population | Count | Population |
| :---: | :---: | :---: | :---: |
| France | 57565008 | Belgium | 10100631 |
| Germany | 81538603 | Denmark | 5275791 |
| United Kingdom | 57654353 | Ireland | 3375748 |
| Spain | 40003942 | Luxembourg | 402437 |
| Portugal 6 | 9953723 | Greece | 10510996 |
| Italy | 57246023 | Netherlands | 15342761 |

The table below presents the results of European elections in Ireland.

Election of Irish representatives to the European Parliament

| Party A |  | Party B |  | Party C |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Candidate | Number <br> of <br> votes | Candidate | Number <br> of <br> votes | Candidate | Number <br> of <br> votes |
| 1 | 40322 | 1 | 9353 | 1 | 3455 |
| 2 | 52344 | 2 | 23437 | 2 | 63843 |
| 3 | 14030 | 3 | 19436 | 3 | 5462 |
| 4 | 85504 | 4 | 1321 | 4 | 25431 |
| 5 | 22460 | 5 | 41356 | 5 | 7465 |
| 6 | 32570 | 6 | 8665 | 6 | 13767 |
| 7 | 27874 | 7 | 26526 | 7 | 32763 |
| 8 | 19602 | 8 | 51243 | 8 | 45274 |
| 9 | 29420 | 9 | 17249 | 9 | 14387 |
| 10 | 27302 | 10 | 9432 | 10 | 6459 |
| Total: 351428 | Total: 208018 | Total: 218306 |  |  |  |



In addition, the following is assumed:

- Each country is attributed a number of seats that is proportional to its population.
- The population of Ireland elects its European representatives using proportional representation.
- The seats won by a party are awarded to the candidates from that party who received the most votes.

Which candidates represented Ireland in the European parliament?

