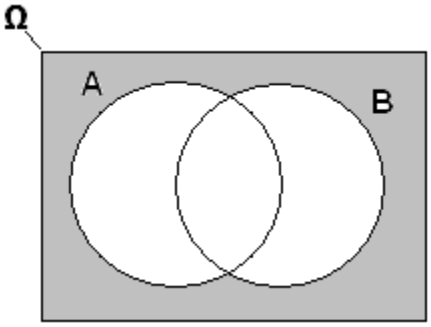
School Year:	2017-2018	D'Arcy McGee High-School	
Math Subject:	**CST** Grade 11 (Cycle 2 / Year 3)		
Evaluation Type:	Competency 2 (Reasoning)		
Scope of the Test:	***Probability, Graph Theory and Optimization***		
Duration: 50 minutes	Mr. Randimbiarison	February 1st, 2018	

NAME: _____**SOLUTIONS**_____ Section MCU504 _____

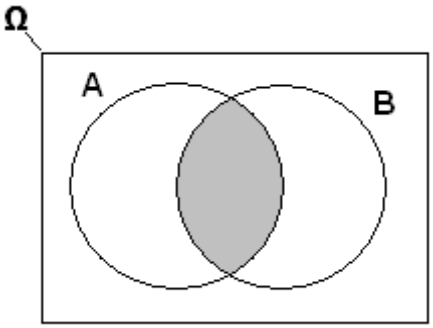
Part A: Multiple Choice Questions (Each question is worth four marks)

1. Which one of the diagrams below illustrates the set (or the event) that is represented by the builder notation **A' ∩ B**?

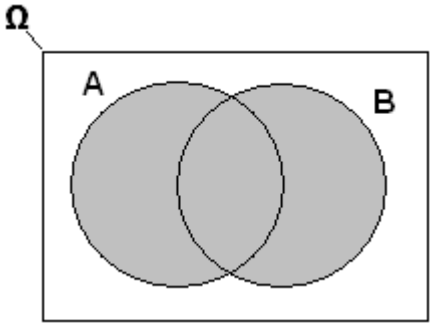
A)



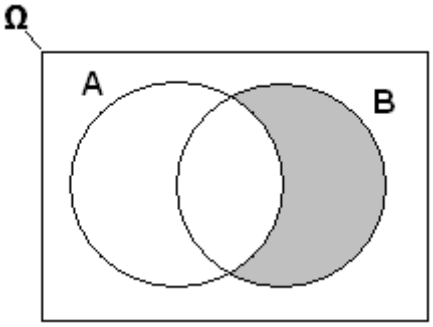
B)



C)



D)



0	4

2. One jar contains 5 red marbles and 3 blue marbles. A second jar contains 2 red and 4 blue marbles. You randomly pick one marble from each jar. What is the probability for picking 2 blue marbles? Answer: $\frac{3}{8} \times \frac{4}{6} = \frac{12}{48}$ or $\frac{1}{4}$

A) $\frac{1}{2}$

B) $\frac{6}{7}$

C) $\frac{1}{4}$

D) $\frac{3}{10}$

0	4

3. There are 14 students in Mr. Tremblay's class. Five are girls and the rest are boys. Two students are called in randomly by the Principal to pick up some learning materials for Mr. Tremblay's class. What are the odds against choosing two boys?

Answer: P(Boy and Boy) = $\frac{9}{14} \times \frac{8}{13} = \frac{72}{182}$

→ Odds for choosing 2 boys = 72: (182- 72) → 72:110

→ Odds against choosing 2 boys are 110:72

A) 17:10

C) 72:110

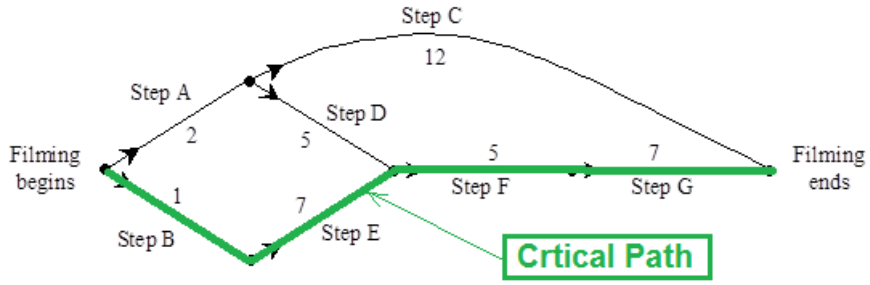
B) 10:17

D) 110:72

0	4

4. The following graph represents the different steps involved in filming a movie. Several steps can be carried out at the same time. The number on each edge indicates the number of days needed to complete the corresponding step.

Answer:



Which one of the numbers below represents the minimum time needed to film the movie?

A) 14

C) 19

B) 20

D) 27

0	4

5. John has red fish (x) and moon fish (y) in his aquarium.

To maintain a healthy environment for the two types of fish, John must respect the constraints below:

$x \geq 0$

$x \geq 2$ *(shade to the right of the vertical line x=2)*

$y \geq 0$

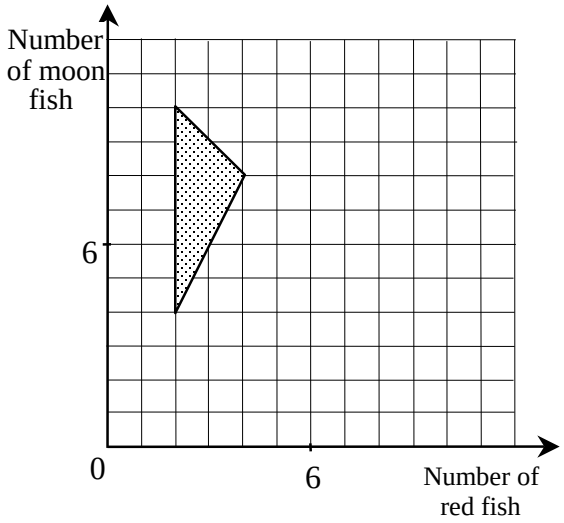
$y \leq 8$ *(shade below the horizontal line y = 8)*

$x + y \leq 12 \rightarrow y \leq -x + 12$ *(shade below line y =-x + 12)*

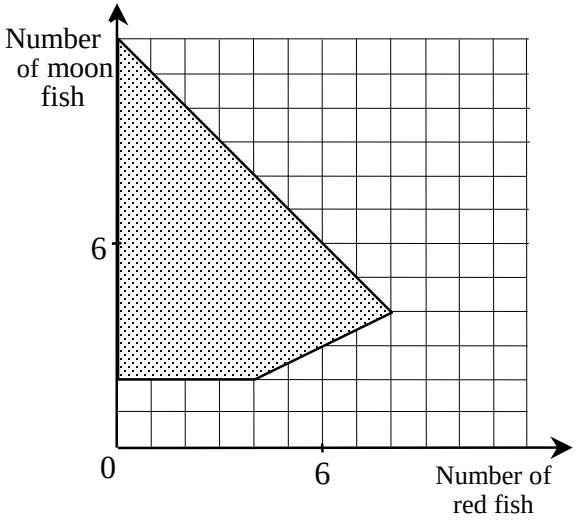
$x \leq 2y \rightarrow y \geq \frac{x}{2}$ *(shade above line y =$\frac{x}{2}$)*

Which of the following polygons of constraints represents the situation above?

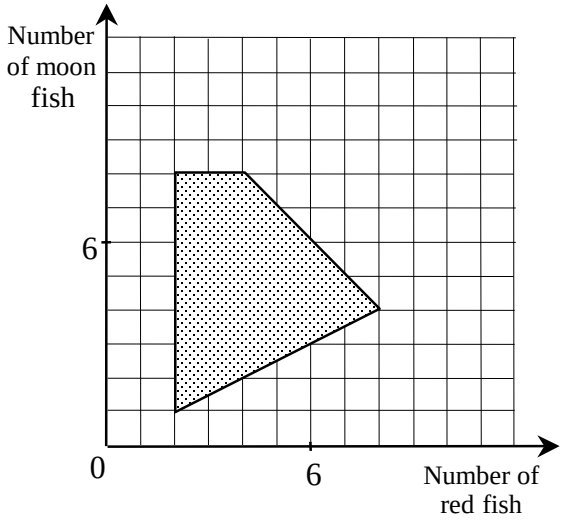
A)



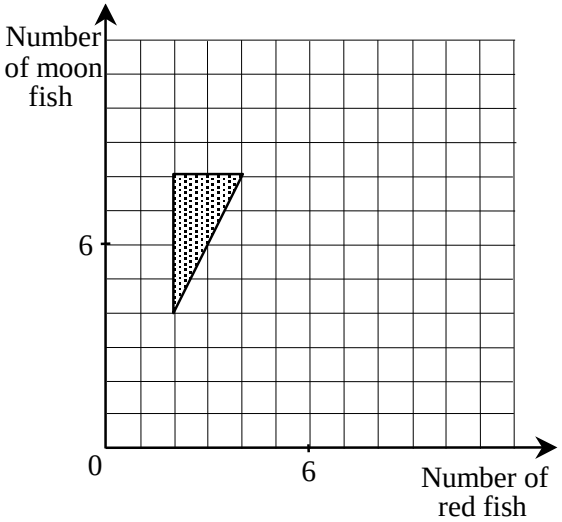
C)



B)



D)

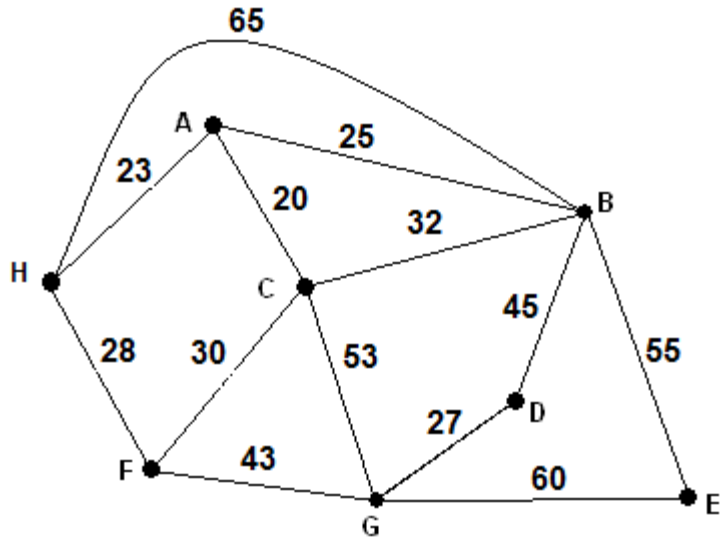


0	4

Part B: Short Answer Questions (Each question is worth four marks)

Note: Detailed work will not be evaluated, i.e. only results will be marked.

6. Given the graph below.



Determine *d (H, D).*

Answer: The distance *d (H, D)* = 2 (edge H-B + edge B-D)

0	4

7. A probability situation involving three events A, B and Cis summarized in the Venn diagram below. Answer: #Ω=45+30+50+25+10+40+70+65 =335
#A = 45+30+10+25=110
(AUB)' = 65 + 70 = 135

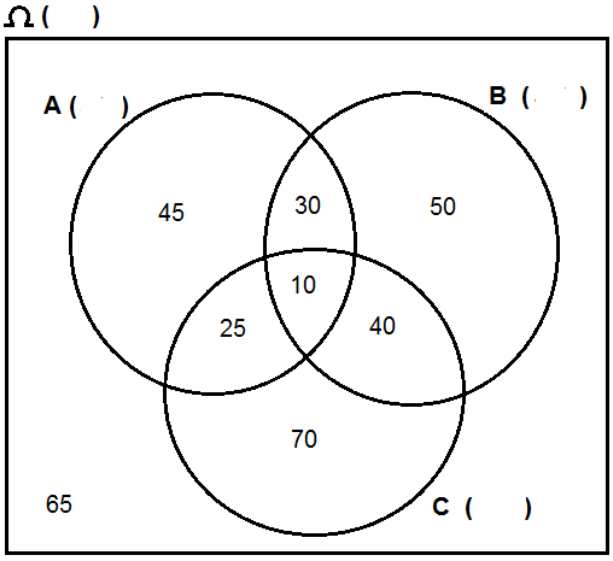
Calculate the following probabilities:

a) P(C|A) = $\frac{25+10}{110}$

Answer: $\frac{35}{110}$ or $\frac{7}{22}$

b) P((AUB)')

Answer: : $\frac{135}{335}$ or $\frac{27}{67}$



0	2	4

8. A game consists of rolling a six-sided die followed by a draw of a marble from a box that contains 3 blue marbles and 5 red marbles. Calculate the probability of rolling a 1 or a 6 from the die followed by a draw of a blue marble.

Solution:

P (1 or 6) = $\frac{2}{6}$;

P(Blue marble) = $\frac{3}{8}$

→ P ((1 or 6) AND Blue marble) = $\frac{2}{6} \times \frac{3}{8} = \frac{6}{48}$

Answer: The probability of rolling a 1 or a 6 followed by a draw of a blue marble is $\frac{6}{48}$ or $\frac{1}{8}$

0	4

Part C: Long Answer Questions (Each question is worth 10 marks)

9. Tickets are on sale for the presentation of *Romeo and Juliet* at Laurentian Regional High School. Tickets cost $15 per student and $20 per adult. The auditorium can seat 450 people. The school hopes to sell at least $6000 worth of tickets. The number of student tickets sold is always greater than or equal to twice the number of adult tickets sold.

The school makes a profit of $10 per student ticket sold and $15 per adult ticket sold.

Because tickets sales have been much better than expected, extra seats have been added, bringing the total number up to 600.

Let: *x* is the number of student tickets sold,
y is the number of adult tickets sold.

By how much will the school's maximum profit increase because of the change in ticket sales? Show your work.

Show your work

Step 1: The objective is to maximize the profit by selling student and adult tickets
Step 2: Optimizing function: Profit ($), P = 10 x + 15 y
Step 3: Constraints :
a) $x \geq 0$ and $y \geq 0$
b) $15x + 20y \geq 6000$
c) $x \geq 2y$
d) $x + y \leq 450$

Step 4: Graphing:
b) → $y \leq \frac{-15}{20}x + \frac{6000}{20}$ → $y \leq \frac{-3}{4}x + 300$ → Y-int = 300, X-int = $\frac{-b}{a} = \frac{-300}{(-\frac{3}{4})}$ =
→ X-int = 400
c) → $2y \leq x$ or $y \leq \frac{x}{2}$ → slope = 2 → UP 1 OVER 2, starting from the origin O (0,0),
→ Shade below line
d) → $y \leq -x + 450$ → slope = - 1 → DOWN 1 OVER 1, starting from b = 450
→ Shade below line

Boundary line (b): $y = \frac{-3}{4}x + 300$
Boundary line (c): $y = \frac{x}{2}$
Boundary line (d) $y = -x + 450$

Scenario 1: #seats ≤ 450
The feasible solutions are represented by the polygon of constraints A, B, C, D.

Since vertex A and vertex D are closer to the origin (0, 0), they will not maximize the profit. Therefore, no need to calculate their coordinates.

Coordinates of the vertices B and C are worth analyzing.

Vertex B:

Intersection of $y = \frac{x}{2}$ and

$y = -x + 450$

$\rightarrow \frac{x}{2} = -x + 450$

$\rightarrow 0.5x + x = 450$

$x = 300 \rightarrow y = 150$

Vertex B (300, 150)

Vertex C:

Intersection of $y = 0$ and

$y = -x + 450$

$\rightarrow x = 450$

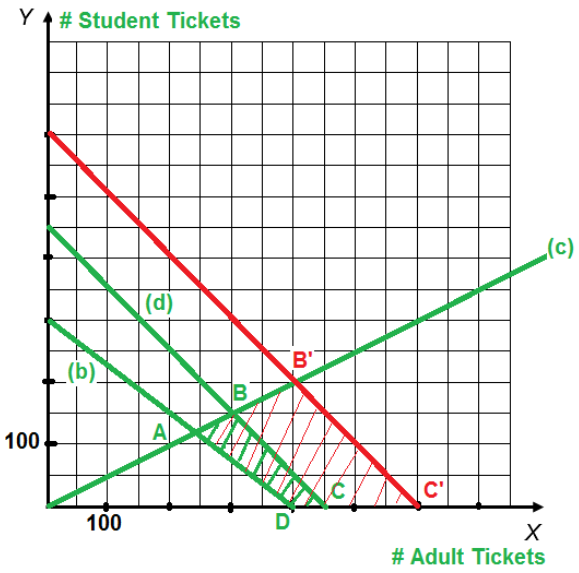
$x = 450 \rightarrow y = 0$

Vertex C (450, 0)

Step 5: Find the Maximum profit

B(300, 150): P = 10(300) + 15(150) = $5250 ← MAX

C(450, 0): P = 10(450) + 15(0) = $4500



Scenario 2: #seats ≤ 600
The feasible solutions are represented by the polygon of constraints A, B', C', D.

Similarly, as seen in scenario 1, B' will maximize the profit.

Vertex B':

Intersection of $y = \frac{x}{2}$ and

$y = -x + 600$

$\rightarrow \frac{x}{2} = -x + 600$

$\rightarrow 0.5x + x = 600$

$x = 400 \rightarrow y = 200$

Vertex B' (400, 200)

B'(400, 200): P = 10(400) + 15(200) = $7000 ←NEW MAX

Therefore, the maximum profit increase is $7000 - $5250 or $1750

Answer: The school's maximum profit increases by $1750

Uses mathematical reasoning							
		Observable indicators correspond to level					
Evaluation Criteria	LEVEL	A	B	C	D	E	
	Cr. 3	40	32	24	16	8	0
	Cr. 2 Cr. 5	40	32	24	16	8	0
	Cr. 4	20	16	12	8	4	0

10. ***The Big Apple Tour*** is a company in New York offering a Manhattan Sight-Seeing Bus tour, an Inside Broadway Walking tour and a Midtown Cruise tours. To attract more tourists, they are offering special prices for visitors:

Visitor Specials	
Manhattan Bus tour	$25/day
Inside Broadway Walking tour	$35/day
Midtown Cruise tour	$40 /day

Tourists who choose to bundle two tours will receive a 15% discount, and those choosing to bundle all three tours will receive a 25% discount.

- a) 54 D'Arcy students have subscribed to at least one of the tours.
- b) 30 have signed up for the Midtown Cruise tour
- c) 12 have signed up for only the Inside Broadway Walking tour
- d) 20 have signed up for two or more tours

If one of the subscribed students is chosen at random:

- e) the probability that they subscribed to all 3 tours is 1/9
- f) the probability that a student subscribed to the Manhattan Bus tour, given that they signed up for the Midtown Cruise tour is 1/3
- g) the probability that they have subscribed to the Manhattan Bus tour and to the Inside Broadway Walking tour only, given that they have gone to two or more tours is 10%

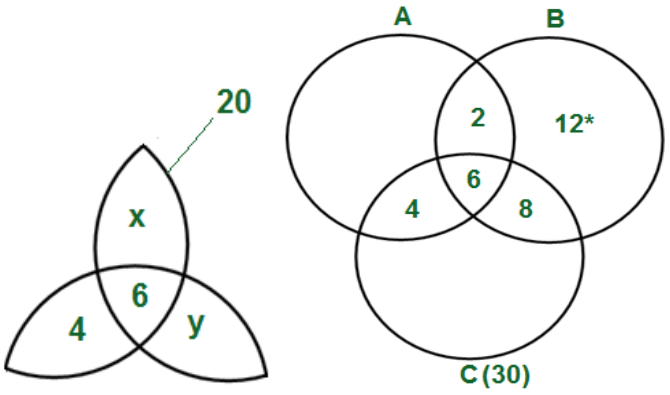
The Big Apple Tour wants to encourage more students to bundle by showing them how much they could save with the bundle discounts.

What is the average daily savings by a student who chooses to go for two or more tours?

A: subscribes for the Manhattan Bus tour
B: subscribes for the Inside Broadway Walking tour
C: subscribes for the Midtown Cruise tour

Show your work

Answer:



a)#Ω = 54 students each must subscribe at least to one tour
b) Number of the students who went for the Midtown Cruise tour is 30 → #C = 30
c) 12 subscribed for Inside Broadway Walking ONLY*
d) #(A∩ B) ∪ (B∩ C) ∪ (A ∩ C) = 20 ← represented by intersecting regions on the right.
e) P(A∩ B ∩ C) = $\frac{1}{9}$ → #A∩ B ∩ C = $\frac{54}{9}$ = 6
f) P(A|C) = $\frac{1}{3}$ → # A ∩ C= $\frac{30}{3}$ = 10 or 6 + 4
g) the probability that students (x) have subscribed to the **Manhattan Bus tour and to the Inside Broadway Walking tour ONLY**, given that they have gone to two or more tours is 10% → 0.10 = $\frac{x}{20}$
→ x = 2
→ y = 20 – (6+4+2) = 8

Average daily savings for a student who chooses to go on a two or more tours:

2 students x ($25 + $35) x 15% discount = $18
4 students x ($25 + $40) x 15% discount = $39
8 students x ($35 + $40) x 15% discount = $90
6 students x ($25 + $35 + $40) x 25% discount = $150

Total discount for 20 students who choose to bundle two or more tours is:
$18 + $39 + $90 + $150 = $297

Average daily savings for a student who chooses to bundle two or more tours is:
$\frac{\$297}{20}$ = $14.85

The average savings by a student who chooses to go for two or more tours is

$14.85_

Uses mathematical reasoning							
		Observable indicators correspond to level					
Evaluation Criteria	LEVEL	A	B	C	D	E	
	Cr. 3	40	32	24	16	8	0
	Cr. 2 Cr. 5	40	32	24	16	8	0
	Cr. 4	20	16	12	8	4	0

11. A study of the impact of flu shots on the health of the population of Gatineau was conducted in the region last year. The total number of people in the sample was 1510. The results of the study is shown in the table below.

	The person had a flu shot **(S)**	The person did not have a flu shot **(NoS)**	Total
The person catches the H1N1 virus **(V)**	a = 50	b = 560	C = 610
The person did not catch the H1N1 virus **(NoV)**	d= 90	e = 810	f= 900
Total	g= 140	1370	1510

- 1) The probability that a person caught the H1N1 virus given that he/she had a flu shot is 5/14
- 2) The probability that a person had a flu shot given that he/she did not catch the H1N1 virus is 10%

Complete the table above and calculate the probability that a person caught the H1N1 virus given that he/she did not have a flu shot.

Show your work

g= 1510 – 1370 = 140

1) → P(V|S) = $\frac{5}{14}$ → #(V ∩ F) = $\frac{5}{14}$ x 140 = 50 → d = 90

2) → P(S|NoV) = 10% → $\frac{90}{\#S}$ = 0.10→ #S = $\frac{90}{0.10}$ = 900 →f = 900

c = 1510 – 900 → c = 610

b = 610 – 50 = 560

e = 900 – 90 = 810

➔ P(V|NoS) = $\frac{560}{1370}$ or $\frac{56}{137}$ or about 41%

The probability that a person caught the H1N1 virus given that he/she did not have a flu shot is

$$\frac{560}{1370}$$

Uses mathematical reasoning							
		Observable indicators correspond to level					
Evaluation Criteria	LEVEL	A	B	C	D	E	
	Cr. 3	40	32	24	16	8	0
	Cr. 2 Cr. 5	40	32	24	16	8	0
	Cr. 4	20	16	12	8	4	0

12. The world's famous gambler from Las Vegas, Señor Pedro, proposes the following game of chance to you. You roll a six-sided fair die. If you roll a 1, then Señor Pedro pays you $25. If you roll a 2, Señor Pedro pays you $5. If you roll a 3, you must pay Señor Pedro $10. If you roll a 4 or a 5, you win $1, and if you roll a 6, you must pay Señor Pedro $20. Is this game favorable to you or to Señor Pedro? Explain.

Show your work

E.V = $25\left(\frac{1}{6}\right) + 5\left(\frac{1}{6}\right) + -10\left(\frac{1}{6}\right) + 1\left(\frac{2}{6}\right) + -20\left(\frac{1}{6}\right)$

E.V = $\left(\frac{25+5-10+2-20}{6}\right)$

E.V = $\left(\frac{2}{6}\right) > 0$

Conclusion: Because the expected value is positive, E.V. > 0, the game is favorable to me, the player, playing against Señor Pedro

Explanation: The game is favorable to any player playing against Señor Pedro because the Expected Value is positive.

Uses mathematical reasoning							
		Observable indicators correspond to level					
Evaluation Criteria	LEVEL	A	B	C	D	E	
	Cr. 3	40	32	24	16	8	0
	Cr. 2 Cr. 5	40	32	24	16	8	0
	Cr. 4	20	16	12	8	4	0