| CST11 Math | Assignment / homework | MCU504 |  |
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| Mid Year - Review Booklet | Probability - Graph Theory - Optimization |  |  |
| Teacher: Mr. Randimbiarison | January, 2018 |  |  |

NAME: $\qquad$ SECTION: MCU504 $\qquad$

## SECTION 1: Probability (Assignment/Test and Midyear Review)

## Short answers: Each sub-questions for each question is worth zero or 2 marks

1. One jar contains 5 red marbles and 3 blue marbles. A second jar contains 2 red and 4 blue marbles. You randomly pick one marble from each jar.
a) What is the probability of picking 2 red marbles? $\qquad$
b) What is the probability of picking 2 blue marbles? $\qquad$
c) What are the odds against picking 2 red marbles?
d) What are the odds against picking 2 blue marbles? $\qquad$
e) What are the odds of getting a blue then a red marbles? $\qquad$
2. There are 21 students in Mr. Roberts's class. 8 are girls and the rest are boys. The Principal removed two students randomly out of the class to help out for the school concert.
a) What is the probability of picking two girls?
b) What are the odds for picking two boys?
c) What are the odds of picking "a boy then a girl"?
d) What are the odds for picking two girls?
e) What is the probability of picking "a boy then a girl"?
$\qquad$
$\qquad$
$\qquad$

## Long answers: Each question is worth 10 marks. Show detailed and clear answer

3. A game involves drawing cubes of the same size but different colors from a box. There are 6 red cubes, 3 yellows cubes and 1 blue cube. If you draw a:

- RED cube you lose $\$ 5$
- YELLOW cube, you win nothing
- BLUE cube you win $\$ 15$

Mélanie claims that this game is in the player's advantage. Is she right? Explain. (10 marks)
4. A wheel is divided into four sections. Players bet $\$ 3$ and spin the wheel. The bet is not returned. If the wheel lands on yellow, you win nothing. If the wheel lands on red, you win $\$ 5$.
If the wheel lands on blue, you win $\$ 3$.
If the wheel lands on green, you win a certain amount of money.

If the game is fair, how much should you win if the wheel lands on green? (10 marks)

5. A game consists of rolling two six-sided dice. A bet of $\$ 2$ must be paid before playing. If you roll and obtain two identical odd numbers you win $\$ 10$.
If you roll and obtain two identical even numbers you win a certain amount If you roll any other pairs of numbers, you win nothing.

The game is fair.
Tim decided to play and he rolled a pair $(4,4)$. How much will Tim win? (10 marks)
6. A game $\boldsymbol{\Omega}$ is defined as the sample space of a given probability experiment. The shaded regions represent a set of items or probable outcomes. $\mathbf{A}$ and $\mathbf{B}$ are two events of $\boldsymbol{\Omega}$.

For each diagram, fill the blank line below it with one of the ten set-builder notations below, that matches the event represented by the shaded area. The list is:
$A ; A^{\prime} ; B ; B ' ; A \cap B ; A \cup B ; A^{\prime} \cup B ; A^{\prime} \cap B ; B \cup A^{\prime} ;(A \cup B)^{\prime}$
$\Omega$

$\Omega$

$\qquad$

$\qquad$

$\Omega$

$\qquad$
$\qquad$
7. In the diagrams below, $\boldsymbol{\Omega}$ is a sample space of a given probability experiment. A, B and $\mathbf{C}$ are three probable events within $\boldsymbol{\Omega}$. The numbers in the circles represent the tally of items belonging to specific subsets of $\boldsymbol{\Omega}$. Here is a list of seven set-builder notations of probable events within $\boldsymbol{\Omega}$ :
$A \cap(B \cap C) ;(A \cap B) \cup C ; A^{\prime} \cap(B \cup C) ;(A \cup B) \cap C^{\prime} ;(A \cup B) \cup C^{\prime} ;(A \cap B) \cup C^{\prime} ;$
$A \cup(B \cap C)$
From the above list, match the one that corresponds to the event that is represented by the shaded area, in the diagrams below, and determine the probability of that event.


Set-builder notation: $\qquad$
Probability of the event: $\qquad$


Set-builder notation: $\qquad$
Probability of the event: $\qquad$
8. Jacques is conducting a probability experiment, which consists of rolling a 6 -sided regular die once. A and B are two events within the sample space $\boldsymbol{\Omega}$, such that:

A: "getting an even number"
B: "getting a number equal of less than 4 "

a) Represent this situation in a Venn diagram (use the diagram above).
b) Determine $P(A), P(B), P(A \cap B)$, and $P(A U B)$ using information from the completed Venn diagram obtained in a). .
c) The union of two events $A$ and $B$ can be expressed by the general formula

- $P(A \cup B)=P(A)+P(B)-P(A \cap B)$.
- Two events are said to be mutually exclusive if $\mathbf{P}(A \cap B)$ or $A \cap B=\varnothing$ or \# $A \cap B=0$. or $\mathbf{P}(A \cup B)=\mathbf{P}(\mathbf{A})+\mathbf{P}(B)$

Using the formulas and properties above, show if, in the current situation, A and B are either two mutually exclusive events or two non-mutually exclusive events.
d) A box contains two green marbles and seven yellow marbles. A and B are two event events such that:
A: "drawing a green marble"
B: "drawing a yellow marble"
In what scenario will the two events independent. Give an example and justify your answer.
9. Three marbles are drawn successively from an urn. For each draw, the color of the marble is recorded and is kept outside the urn. The urn contains 6 red, 4 green, and 3 blue marbles.
a) Calculate the probability of getting $\mathbf{2}$ green marbles in the first two draws and a red or a blue marble in the third draw.
b) Calculate the probability of getting at least two red marbles.
(Use of the tree diagram below is optional)

10. In one of Mr. Jones classes, there are $\mathbf{3 0}$ grade 11 Math students:

8 students are left-handed.
16 students wear T-shirts.
17 students have brown hair.
4 students with brown hair wear T-shirts and are left-handed.
19 students are left-handed or have brown hair.
9 students have brown hair and wear T-shirts.
24 students wear T-shirts or have brown hair.
5 students wear T-shirts and are left-handed.
A student is chosen at random from this group. The following are 3 possible events:
A: choosing a left-handed students
B: choosing a student who wears T-shirts
C: choosing a student with brown hair
a) Represent this situation using a Venn diagram.
b) Express the following event using a set-builder notation:
"Choosing a student who has brown hair and wears T-shirt or a left-handed student"
c) In this situation, interpret $(A \cup B) \cap C$ in plain English.
d) Calculate $P((A \cup B) \cap C)$

Calculate the probability of choosing a student who does not wear a T-shirt and is not lefthanded.
11. A, B, and C are events (or subsets) of a universal set $\Omega$, where $\#(\Omega)=800$

Givens:
a) $\#(A \cap B \cap C)=15$
b) $P(A)=1 / 4$
c) $P(A \cap B)=1 / 8$
d) $P(C \mid A)=1 / 5$
e) $P[(A \cap B) U(A \cap B) U(B \cap C)]=20 \%$
f) $\#(A \cup B U C)^{\prime}=120$
g) $P(C \mid B)=12.5 \%$
$\Omega($ )


Question: Complete the Venn diagram above and determine $\mathrm{P}(\mathrm{A} \mid \mathrm{C})$.

## 12. A, B, and C are events

(or subsets) of a universal set $\Omega$, where $\#(\Omega)=1200$

## Givens:

a) $P(A \cap B \cap C)=10 \%$
b) $P(B \cap C)=1 / 8$
c) $P(C)=25 \%$
d) $\mathrm{P}[(\mathrm{AUBUC})$ ' $]=5 \%$
e) $\mathrm{P}(\mathrm{A} \mid \mathrm{C})=0.5$
f) $P(A)=30 \%$
g) $P(B \mid A)=1 / 3$


Question:
a) Complete the Venn diagram above.
b) Rachel claims that at least $50 \%$ of $\Omega$ do not have anything in common with A and C and that these 50\% are only in B. Is she right?
c) Determine $\mathrm{P}(\mathrm{C} \mid \mathrm{B})$.
13. $\mathrm{A}, \mathrm{B}$, and C are events (or subsets) of a universal set $\Omega$.

## Givens:

\#(AUBUC)' $=75$
$P(A \mid B)=1 / 8$
$P(C \mid A)=20 \%$
Questions:
a) Complete the Venn diagram on the right.
b) Determine \#( $\Omega$ )
c) Determine $\mathrm{P}\left(\mathrm{C}^{*}\right)$, where $\mathrm{C}^{*}$ represents a set of values that belong exclusively or strictly to C only.

## $\Omega($ )


14. A survey of 1200 students was conducted at this local High School about their involvement in three particular team sports, soccer, basketball and rugby. The survey shows that:

- The probability of selecting at random a student who plays all three sports is 1 / 10
- 80 do not play any of the three-team sports
- 500 play soccer
- $1 / 4$ play rugby
- 220 play soccer and basketball
- The probability of selecting a student who plays soccer and rugby is $1 / 6$
- The probability of selecting a student who plays basketball given that he or she plays rugby is $60 \%$.


## Question:

Draw a Venn diagram for this situation and determine the probability of finding a student who plays rugby given that he/she plays basketball.
15. The Big Apple Tour is a company in New-York offering a Manhattan Sight-Seeing Bus tour, an Inside Broadway Walking tour and a Midtown Cruise tours. To attract more tourists, they are offering special prices for visitors:

| Visitor Specials |  |
| :--- | :--- |
| Manhattan Bus tour | $\$ 25 /$ day |
| Inside Broadway Walking tour | $\$ 35 /$ day |
| Midtown Cruise | $\$ 40 /$ day |
|  |  |

Tourists who choose to bundle two tours will receive a $15 \%$ discount, and those choosing to bundle all three tours will receive a $25 \%$ discount.

- 54 D'Arcy students have subscribed to at least one of the tours.
- 20 have signed up for the Midtown Cruise
- 12 have signed up for only the Inside Broadway Walking tour service
- 30 have signed up for two or more services

If one of the subscribed students is chosen at random:

- the probability that they subscribed to all 3 services is $\mathbf{1 / 9}$
- the probability that a teacher subscribed to the Manhattan Bus tour service, given that they signed up for the Midtown Cruise service is $1 / 3$
- the probability that they have subscribed to the Manhattan Bus tour and to the Inside Broadway Walking tour services ONLY, given that they have bundled two or more services is $10 \%$

The Big Apple Tour wants to encourage more students to bundle by showing them how much they could save with the bundle discounts.
What is the average monthly savings by a student who chose to bundle two or more tours?
A: subscribes for the Manhattan Bus tour
B: subscribes for the Inside Broadway Walking tour
C: subscribes for the Midtown Cruise

16. A school organized a ski trip for Secondary $V$ students. The following table shows the distribution of students on that trip.

|  | Girls | Boys |
| :--- | :---: | :---: |
| Went snowboarding | 205 | 188 |
| Went downhill skiing | 36 | 41 |
| Went cross-country skiing | 17 | 11 |

A girl was selected at random from among the students who went on the trip.
a) What is the probability of selecting a girl who went downhill skiing?
b) Given that the student went snowboarding, what is the probability that it is a boy?

A study of speeding violations and drivers who use car phones produced the following data. The total number of people in the sample is 755 .

|  | Speeding violation <br> in the last year | No speeding violation <br> in the last year | Total |
| :--- | :--- | :--- | :--- |
| Car phone user |  |  |  |
| Not a car phone user | 45 | 685 | 755 |
| Total |  |  |  |

- The probability of finding a person had a speeding violation last year given that he/she was not a car phone user is $10 \%$
- The probability of finding a person who was a car phone user given that he/she had a speeding violation last year is $5 / 14$

Using the information above, fill the table and calculate the following probabilities
a) P (person is a car phone user)
b) P (person had no violation in the last year)
c) P (person had no violation in the last year AND was a car phone user)
d) P (person is a car phone user OR person had no violation in the last year)
e) P (person had no violation last year $G$ person was not a car phone user)

## 18. SAVE A bundle

Smart Media is a new company offering Home Phone, High-Speed Internet and Digital Cable TV services. To attract more teachers to switch to their company, they are offering special prices for educators:

| Educator Specials |  |
| :--- | :--- |
| Home Phone | $\$ 25 /$ month |
| High-Speed Internet | $\$ 30 /$ month |
| Digital Cable TV | $\$ 50 /$ month |
|  |  |

Teachers who choose to bundle two services will receive a $10 \%$ discount, and those choosing to bundle all three services will receive a $20 \%$ discount.

Of the staff at D'Arcy McGee and Symmes schools:

- 48 have subscribed to at least one of the services
- 30 have signed up for the Digital Cable TV service
- 10 have signed up for only the High-Speed Internet service
- 20 have bundled two or more services

If one of the subscribed teachers is chosen at random:

- the probability that they subscribed to all 3 services is $1 / 6$
- the probability that a teacher subscribed to the Home Phone service, given that they signed up for the Digital Cable TV service is $1 / 3$
- the probability that they have subscribed to the Home Phone and High-Speed Internet services ONLY, given that they have bundled two or more services is $10 \%$

Smart Media wants to encourage more teachers to bundle by showing them how much they could save with the bundle discounts.

What is the average monthly savings by a teacher who chooses to bundle two or more service?

H: subscribes to Home Phone
I: subscribes to High-Speed Internet
C: subscribes to Digital Cable TV


## 19. Valid Voting

The student government at a local high school is planning an end of the year carnival for the student body. Three activities there will be sports competitions. The sports facilities at the school allows for the possibly of soccer, rugby, baseball and lacrosse. However, the budget only allows for two of the sporting competitions to take place.

The executive could not decide on which sports to include so they asked all the students in the school to vote. Students were given a ballot listing the four sports and asked to indicate his or her first, second, third and fourth choice with no ties allowed.

## Preference Schedule for the sports voting

| Number of students <br> (who ranked the sport in the <br> order shown) | $\mathbf{1 8 0}$ | $\mathbf{2 2 0}$ | $\mathbf{1 5 0}$ | $\mathbf{1 6 0}$ |
| :--- | :---: | :---: | :---: | :---: |
| First choice | soccer | rugby | baseball | lacrosse |
| Second choice | baseball | soccer | lacrosse | soccer |
| Third choice | rugby | lacrosse | rugby | baseball |
| Fourth choice | lacrosse | baseball | soccer | rugby |

The student government, looking at the results, said that rugby and soccer were the top choices.

Using at least 3 voting procedures, prove whether or not rugby and soccer are the top two preferences of the student population.
Show your work

## More Voting Procedures Questions:

Visions 2 Book:
p. 113 \#1, \#2
p.114, \#4
p.115, \#6
p.116, \#8

