

CST11 Math	Assignment / homework	MCU504	
Mid Year - Review Booklet	Probability – Graph Theory - Optimization		
Teacher: Mr. Randimbiarison		January, 2018	

NAME: _____

SECTION: MCU504_____

SECTION 1: Probability (Assignment/Test and Midyear Review)

Short answers: Each sub-questions for each question is worth zero or 2 marks

1. One jar contains 5 red marbles and 3 blue marbles. A second jar contains 2 red and 4 blue marbles. You randomly pick one marble from each jar.

- a) What is the probability of picking 2 red marbles? _____ /2
- b) What is the probability of picking 2 blue marbles? _____ /2
- c) What are the odds against picking 2 red marbles? _____ /2
- d) What are the odds against picking 2 blue marbles? _____ /2
- e) What are the odds of getting a blue then a red marbles? _____ /2

2. There are 21 students in Mr. Roberts’s class. 8 are girls and the rest are boys. The Principal removed two students randomly out of the class to help out for the school concert.

- a) What is the probability of picking two girls? _____ /2
- b) What are the odds for picking two boys? _____ /2
- c) What are the odds of picking “a boy then a girl”? _____ /2
- d) What are the odds for picking two girls? _____ /2
- e) What is the probability of picking “a boy then a girl”? _____ /2

Long answers: Each question is worth 10 marks. Show detailed and clear answer

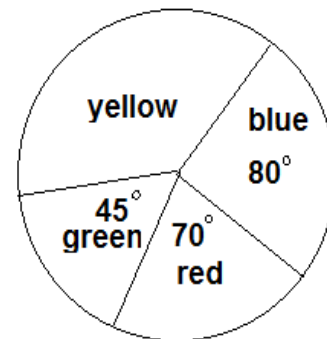
3. A game involves drawing cubes of the same size but different colors from a box. There are 6 red cubes, 3 yellow cubes and 1 blue cube. If you draw a:

- RED cube you lose \$5
- YELLOW cube, you win nothing
- BLUE cube you win \$15

Mélanie claims that this game is in the player's advantage. Is she right? Explain. (10 marks)

4. A wheel is divided into four sections. Players bet \$3 and spin the wheel. The bet is not returned.
If the wheel lands on yellow, you win nothing.
If the wheel lands on red, you win \$5.
If the wheel lands on blue, you win \$3.
If the wheel lands on green, you win a certain amount of money.

If the game is fair, how much should you win if the wheel lands on green? (10 marks)



5. A game consists of rolling two six-sided dice. A bet of \$2 must be paid before playing.
If you roll and obtain two identical odd numbers you win \$10.
If you roll and obtain two identical even numbers you win a certain amount
If you roll any other pairs of numbers, you win nothing.

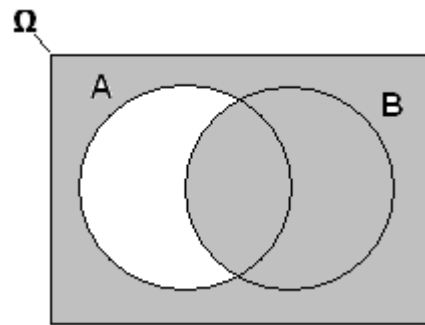
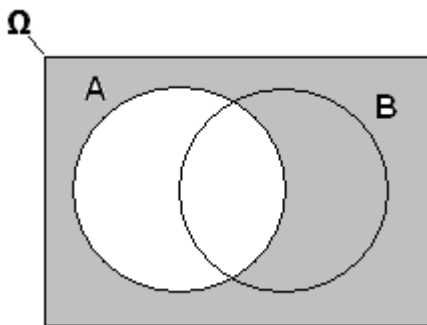
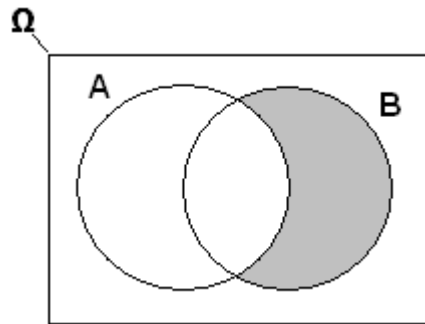
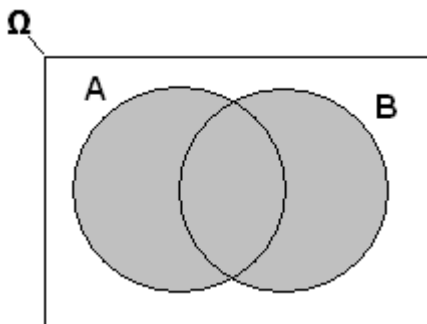
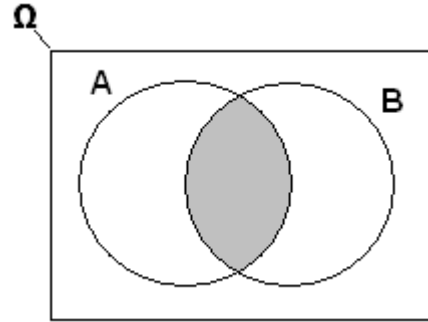
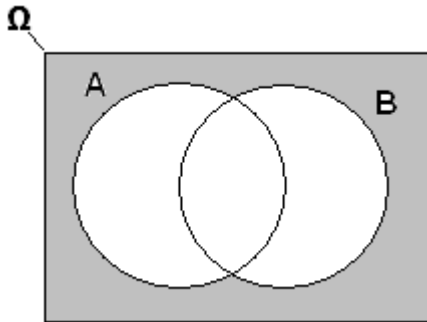
The game is fair.

Tim decided to play and he rolled a pair (4, 4). How much will Tim win? (10 marks)

6. A game Ω is defined as the *sample space* of a given probability experiment. The shaded regions represent a set of items or probable outcomes. **A** and **B** are two *events* of Ω .

For each diagram, fill the blank line below it with one of the ten set-builder notations below, that matches the event represented by the shaded area. The list is:

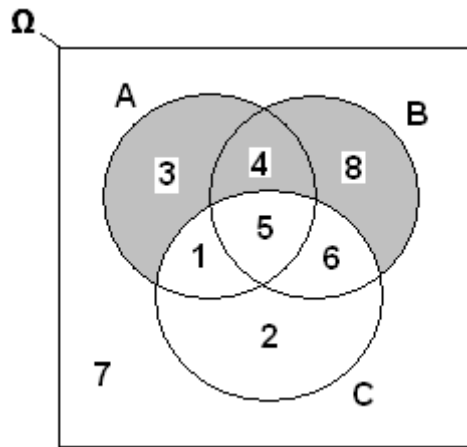
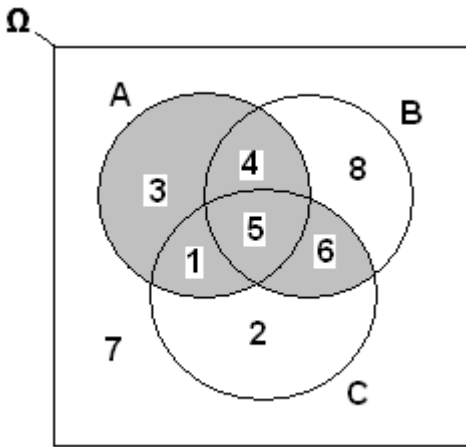
A; A'; B; B'; $A \cap B$; $A \cup B$; $A' \cup B$; $A' \cap B$; $B \cup A'$; $(A \cup B)'$



7. In the diagrams below, Ω is a *sample space* of a given probability experiment. **A**, **B** and **C** are three probable events within Ω . The numbers in the circles represent the tally of items belonging to specific subsets of Ω . Here is a list of seven set-builder notations of probable events within Ω :

- $A \cap (B \cap C)$ $(A \cap B) \cup C$ $A' \cap (B \cup C)$ $(A \cup B) \cap C$ $(A \cup B) \cup C'$
 $(A \cap B) \cup C'$ $A \cup (B \cap C)$

From the above list, match the one that corresponds to the event that is represented by the shaded area, in the diagrams below, and determine the probability of that event.



Set-builder notation:

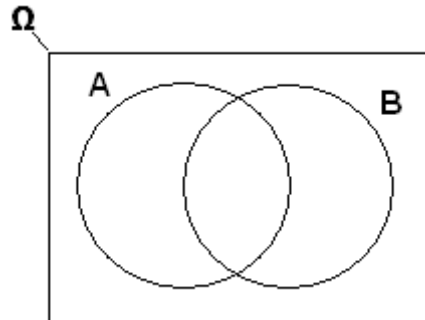
Set-builder notation:

Probability of the shaded event:

Probability of the shaded event:

8. Jacques is conducting a probability experiment, which consists of rolling a 6-sided regular die once. A and B are two events within the sample space Ω , such that:

- A:** “getting an even number”
B: “getting a number equal of less than 4”



- a) Represent this situation in a Venn diagram (use the diagram above).
 b) Determine $P(A)$, $P(B)$, $P(A \cap B)$, and $P(A \cup B)$ using information from the completed Venn diagram obtained in a).

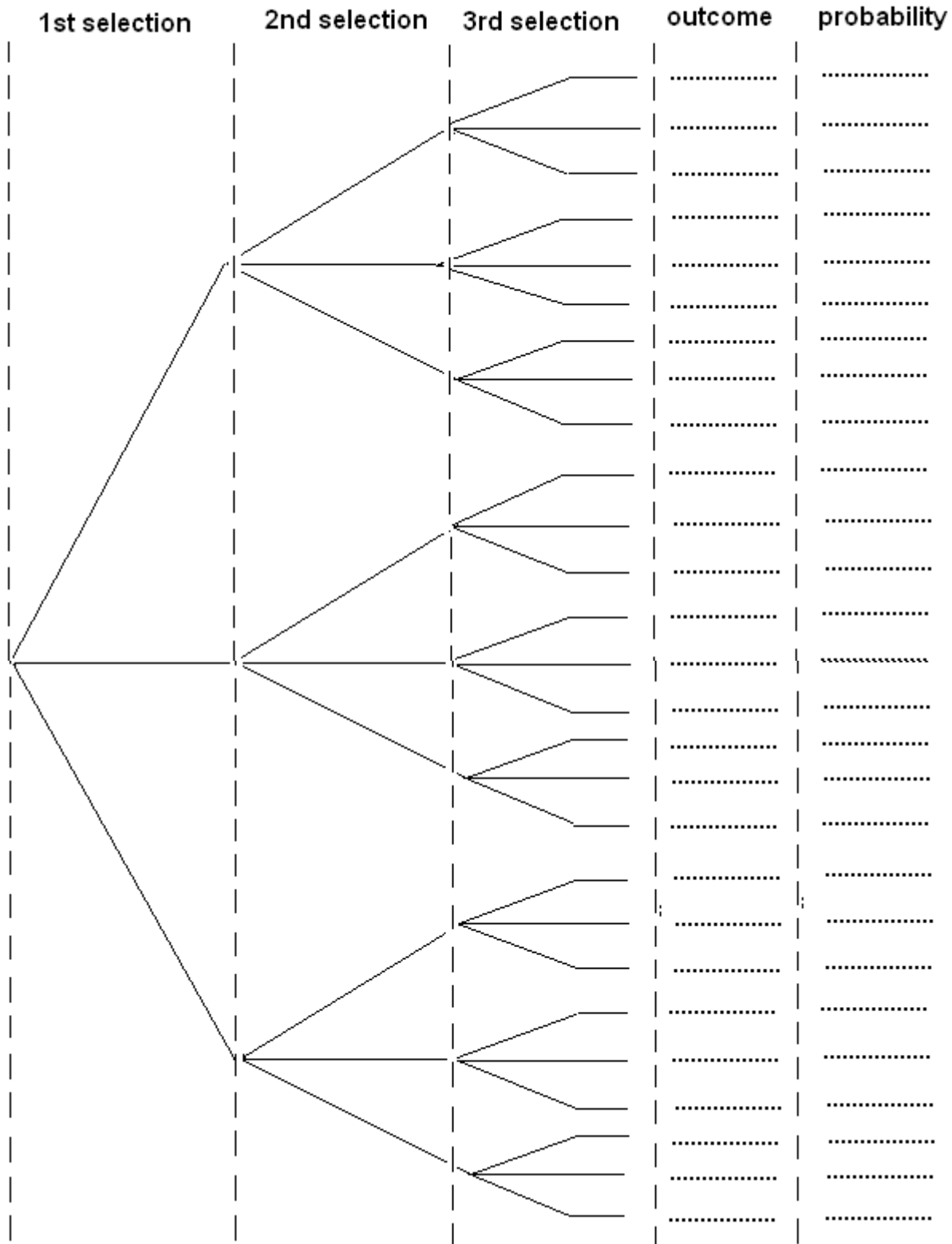
- c) The union of two events A and B can be expressed by the general formula
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
 - Two events are said to be *mutually exclusive* if $P(A \cap B) = 0$ or $A \cap B = \emptyset$ or $\# A \cap B = 0$.
 or $P(A \cup B) = P(A) + P(B)$

Using the formulas and properties above, show if, in the current situation, A and B are either two mutually exclusive events or two non-mutually exclusive events.

- d) A box contains two green marbles and seven yellow marbles. A and B are two event events such that:
A: “drawing a green marble”
B: “drawing a yellow marble”
 In what scenario will the two events independent. Give an example and justify your answer.

- 9.** Three marbles are drawn successively from an urn. For each draw, the color of the marble is recorded and is kept outside the urn. The urn contains 6 red, 4 green, and 3 blue marbles.
- a) Calculate the probability of getting 2 green marbles in the first two draws and a red or a blue marble in the third draw.
- b) Calculate the probability of getting at least two red marbles.

(Use of the tree diagram below is optional)



10. In one of Mr. Jones classes, there are **30** grade 11 Math students:

8 students are left-handed.

16 students wear T-shirts.

17 students have brown hair.

4 students with brown hair wear T-shirts and are left-handed.

19 students are left-handed or have brown hair.

9 students have brown hair and wear T-shirts.

24 students wear T-shirts or have brown hair.

5 students wear T-shirts and are left-handed.

A student is chosen at random from this group. The following are 3 possible events:

A: choosing a left-handed students

B: choosing a student who wears T-shirts

C: choosing a student with brown hair

a) Represent this situation using a Venn diagram.

b) Express the following event using a set-builder notation:

“Choosing a student who has brown hair and wears T-shirt or a left-handed student”

c) In this situation, interpret $(A \cup B) \cap C$ in plain English.

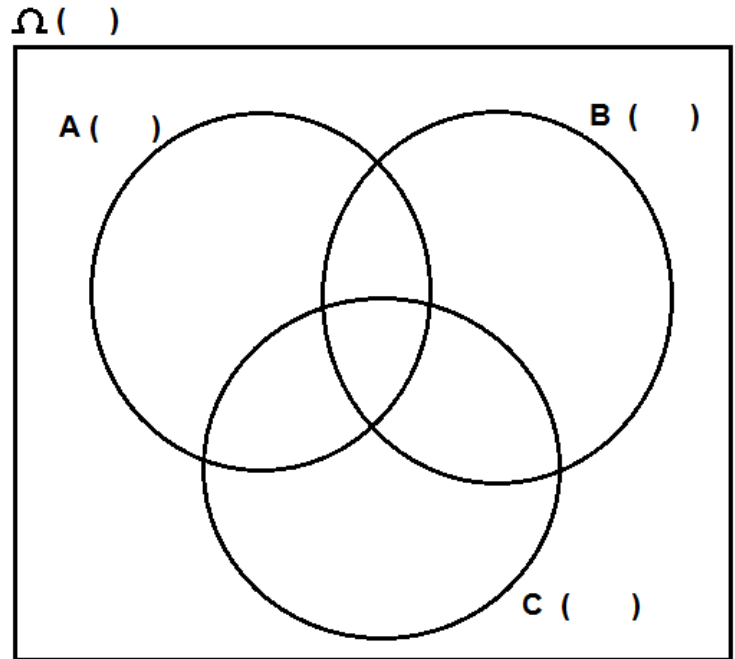
d) Calculate $P((A \cup B) \cap C)$

Calculate the probability of choosing a student who does not wear a T-shirt and is not left-handed.

11. A, B, and C are events (or subsets) of a universal set Ω , where $\#\Omega = 800$

Givens:

- a) $\#(A \cap B \cap C) = 15$
- b) $P(A) = 1/4$
- c) $P(A \cap B) = 1/8$
- d) $P(C|A) = 1/5$
- e) $P[(A \cap B) \cup (A \cap C) \cup (B \cap C)] = 20\%$
- f) $\#(A \cup B \cup C)' = 120$
- g) $P(C|B) = 12.5\%$

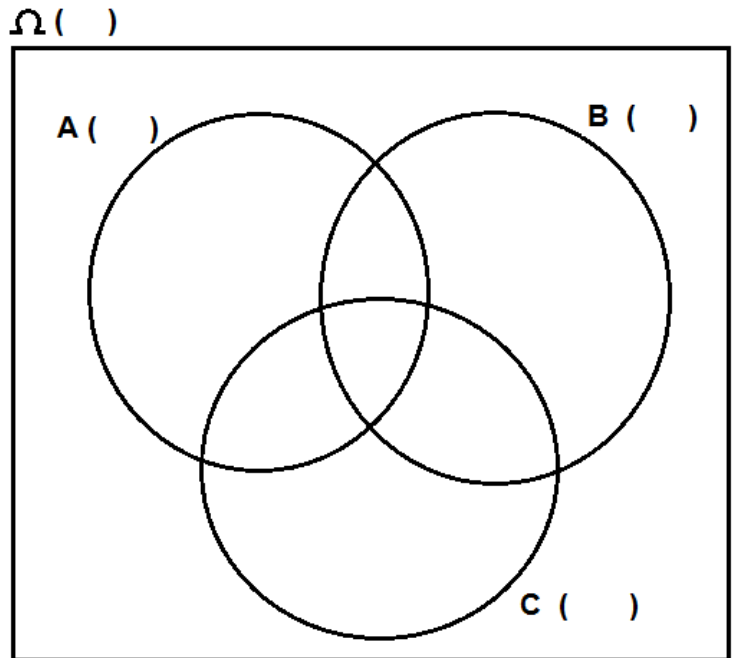


Question: Complete the Venn diagram above and determine $P(A|C)$.

12. A, B, and C are events (or subsets) of a universal set Ω , where $\#\Omega = 1200$

Givens:

- a) $P(A \cap B \cap C) = 10\%$
- b) $P(B \cap C) = 1/8$
- c) $P(C) = 25\%$
- d) $P[(A \cup B \cup C)^c] = 5\%$
- e) $P(A|C) = 0.5$
- f) $P(A) = 30\%$
- g) $P(B|A) = 1/3$



Question:

- a) Complete the Venn diagram above.
- b) Rachel claims that at least 50% of Ω do not have anything in common with A and C and that these 50% are only in B. Is she right?
- c) Determine $P(C|B)$.

13. A, B, and C are events (or subsets) of a universal set Ω .

Givens:

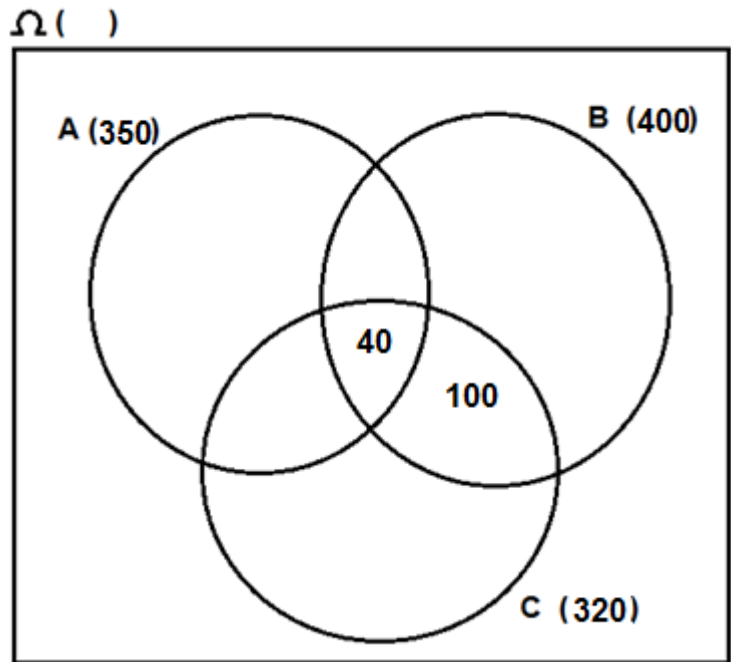
$$\#(A \cup B \cup C)' = 75$$

$$P(A|B) = 1/8$$

$$P(C|A) = 20\%$$

Questions:

- Complete the Venn diagram on the right.
- Determine $\#(\Omega)$
- Determine $P(C^*)$, where C^* represents a set of values that belong exclusively or strictly to C only.



14. A survey of 1200 students was conducted at this local High School about their involvement in three particular team sports, soccer, basketball and rugby. The survey shows that:

- The probability of selecting at random a student who plays all three sports is $1 / 10$
- 80 do not play any of the three-team sports
- 500 play soccer
- $1/4$ play rugby
- 220 play soccer and basketball
- The probability of selecting a student who plays soccer and rugby is $1 / 6$
- The probability of selecting a student who plays basketball given that he or she plays rugby is 60%.

Question:

Draw a Venn diagram for this situation and determine the probability of finding a student who plays rugby given that he/she plays basketball.

15. The Big Apple Tour is a company in New-York offering a Manhattan Sight-Seeing Bus tour, an Inside Broadway Walking tour and a Midtown Cruise tours. To attract more tourists, they are offering special prices for visitors:

<u>Visitor Specials</u>	
Manhattan Bus tour	\$25/day
Inside Broadway Walking tour	\$35/day
Midtown Cruise tour	\$40 /day

Tourists who choose to bundle two tours will receive a 15% discount, and those choosing to bundle all three tours will receive a 25% discount.

- 54 D’Arcy students have subscribed to at least one of the tours.
30 have signed up for the Midtown Cruise tour
- 12 have signed up for only the Inside Broadway Walking tour
- 20 have signed up for two or more tours

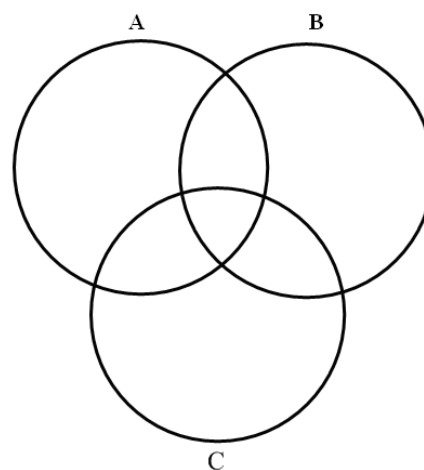
If one of the subscribed students is chosen at random:

- the probability that they subscribed to all 3 tours is $\frac{1}{9}$
- the probability that a student subscribed to the Manhattan Bus tour, given that they signed up for the Midtown Cruise tour is $\frac{1}{3}$
- the probability that they have subscribed to the Manhattan Bus tour and to the Inside Broadway Walking tour ONLY, given that they have bundled two or more tours is 10%

The Big Apple Tour wants to encourage more students to bundle by showing them how much they could save with the bundle discounts.

What is the total amount of daily savings by the group of students who chose to bundle two or more tours?

- A:** subscribes for the Manhattan Bus tour
- B:** subscribes for the Inside Broadway Walking tour
- C:** subscribes for the Midtown tour



16. A school organized a ski trip for Secondary V students. The following table shows the distribution of students on that trip.

	Girls	Boys
Went snowboarding	205	188
Went downhill skiing	36	41
Went cross-country skiing	17	11

A girl was selected at random from among the students who went on the trip.

- a) **What is the probability of selecting a girl who went downhill skiing?**

- b) **Given that the student went snowboarding, what is the probability that it is a boy?**

17. A study of speeding violations and drivers who use car phones produced the following data. The total number of people in the sample is 755.

	Speeding violation in the last year	No speeding violation in the last year	Total
Car phone user			
Not a car phone user	45		
Total		685	755

- The probability of finding a person had a speeding violation last year given that he/she was not a car phone user is 10%
- The probability of finding a person who was a car phone user given that he/she had a speeding violation last year is $5/14$

Using the information above, fill the table and calculate the following probabilities

- P(person is a car phone user)
- P(person had no violation in the last year)
- P(person had no violation in the last year and was a car phone user)
- P(person is a car phone user or person had no violation in the last year)
- P (person had no violation last year given that person was not a car phone user)

18. SAVE A BUNDLE

Smart Media is a new company offering Home Phone, High-Speed Internet and Digital Cable TV services. To attract more teachers to switch to their company, they are offering special prices for educators:

<u>Educator Specials</u>	
Home Phone	\$25/month
High-Speed Internet	\$30/month
Digital Cable TV	\$50 / month

Teachers who choose to bundle two services will receive a 10% discount, and those choosing to bundle all three services will receive a 20% discount.

Of the staff at D'Arcy McGee and Symmes schools:

- 48 have subscribed to at least one of the services
- 30 have signed up for the Digital Cable TV service
- 10 have signed up for only the High-Speed Internet service
- 20 have bundled two or more services

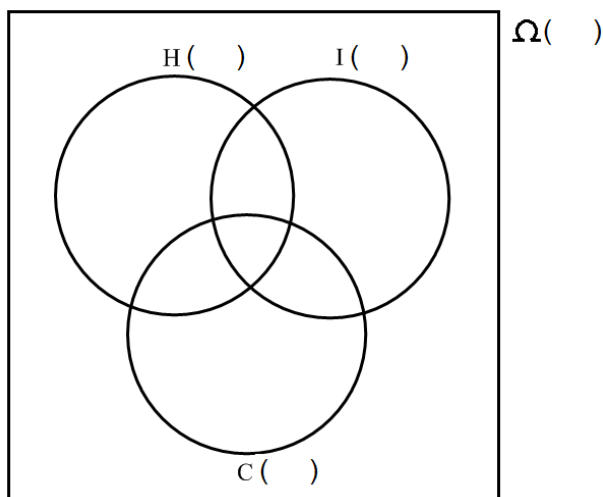
If one of the subscribed teachers is chosen at random:

- the probability that they subscribed to all 3 services is $1/6$
- the probability that a teacher subscribed to the Home Phone service, given that they signed up for the Digital Cable TV service is $1/3$
- the probability that they have subscribed to the Home Phone and High-Speed Internet services ONLY, given that they have bundled two or more services is 10%

Smart Media wants to encourage more teachers to bundle by showing them how much they could save with the bundle discounts.

What is the average monthly savings by a teacher who chooses to bundle two or more service?

H: subscribes to Home Phone
 I: subscribes to High-Speed Internet
 C: subscribes to Digital Cable TV



19. Valid Voting

The student government at a local high school is planning an end of the year carnival for the student body. Three activities there will be sports competitions. The sports facilities at the school allows for the possibly of soccer, rugby, baseball and lacrosse. However, the budget only allows for two of the sporting competitions to take place.

The executive could not decide on which sports to include so they asked all the students in the school to vote. Students were given a ballot listing the four sports and asked to indicate his or her first, second, third and fourth choice with no ties allowed.

Preference Schedule for the sports voting

Number of students (who ranked the sport in the order shown)	180	220	150	160
First choice	soccer	rugby	baseball	lacrosse
Second choice	baseball	soccer	lacrosse	soccer
Third choice	rugby	lacrosse	rugby	baseball
Fourth choice	lacrosse	baseball	soccer	rugby

The student government, looking at the results, said that rugby and soccer were the top choices.

Using at least 3 voting procedures, prove whether or not rugby and soccer are the top two preferences of the student population.

Show your work

More Voting Procedures Questions:

Visions 2 Book:

p.113 #1, #2

p.114, #4

p.115, #6

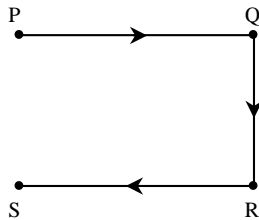
p.116, #8

SECTION 2: Graph Theory

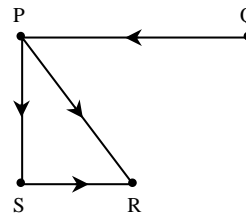
Part A: Fundamental Concepts (Terminologies and basic concepts)

1. Which of the following graphs contains a path of length 4?

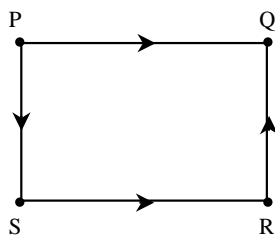
A)



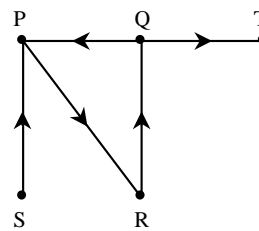
C)



B)

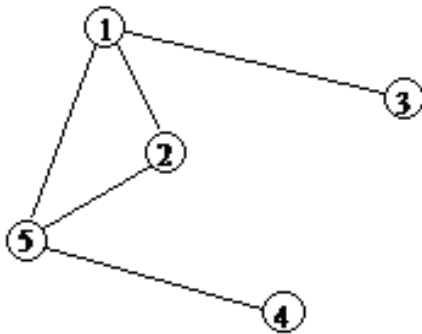


D)

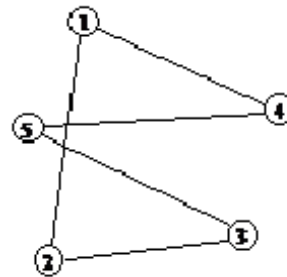


2. Which of the following graphs represents an Euler circuit?

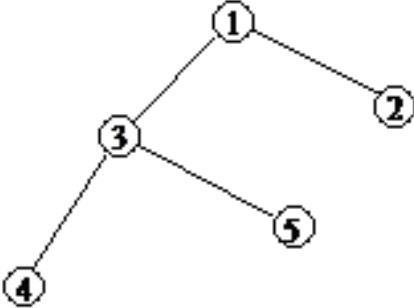
A)



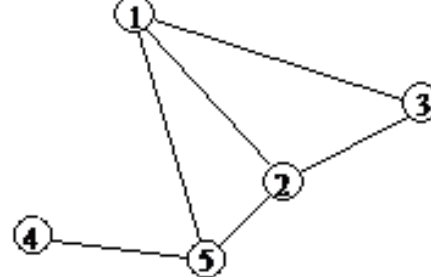
C)



B)



D)



3. Which one of statements below is true?

- A) To obtain an Euler path there must be at least 2 vertices with odd degree.
- B) The number of edges in a complete graph is double the sum of the degrees
- C) An Euler graph is guaranteed when all the degrees of the vertices are even
- D) A tree graph cannot contain a circuit

1. A

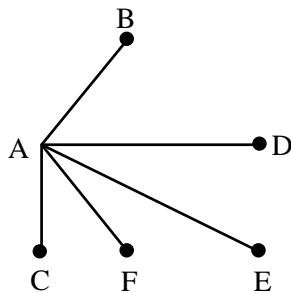
2. B

3. C

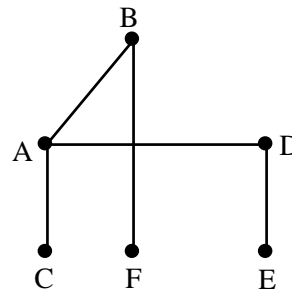
3. D

4. Which of the following graphs contains a circuit that passes no more than once along any given edge?

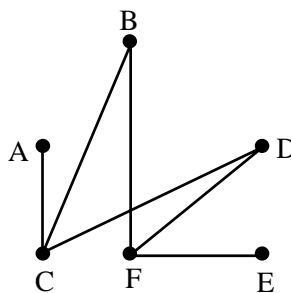
A)



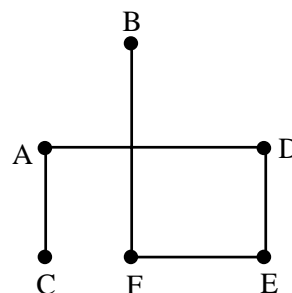
C)



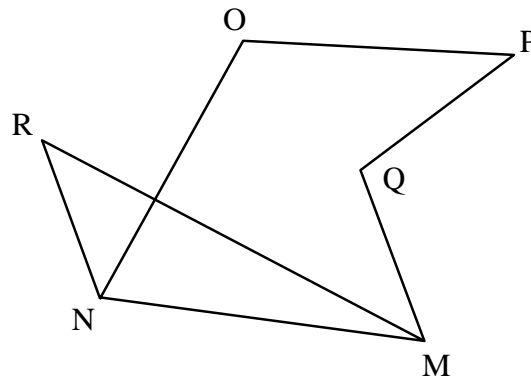
B)



D)



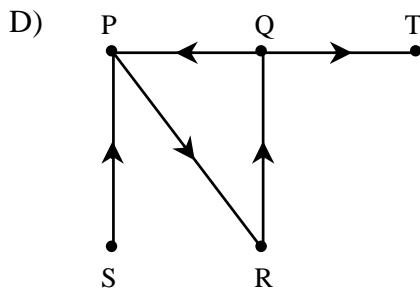
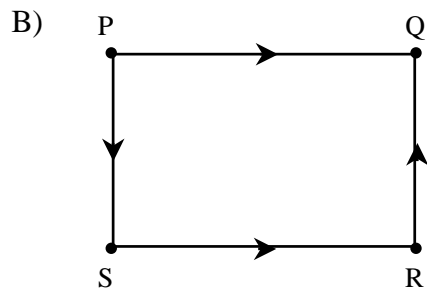
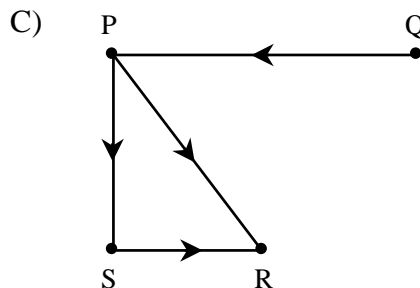
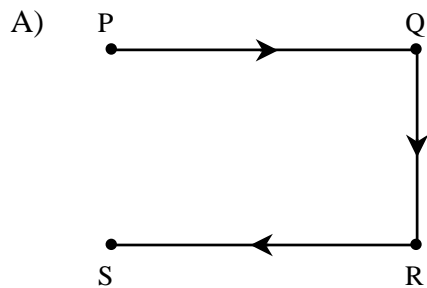
5. Peter wants to change the graph on the right into a tree diagram.



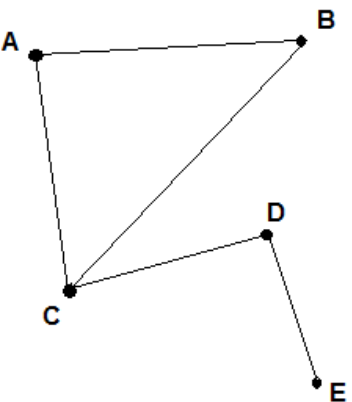
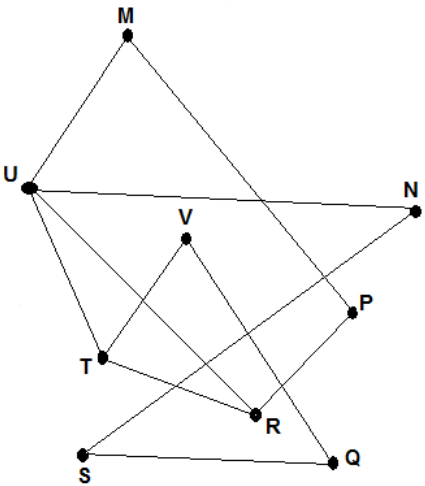
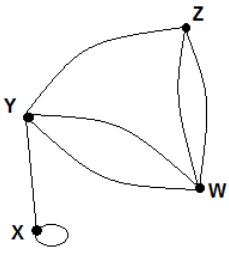
Which of the edges below must be removed to produce a tree diagram?

- A) MR and NR
- B) MQ and NO
- C) MQ and OP
- D) MN and MR

6. Which one of the following graphs contains a circuit?



7. Identify which graphs contain an Euler path. Explain why and specify from which vertex the path starts from and which vertex it ends at.

<p>a)</p>  <p>Euler path exist? YES / NO (circle) Why: _____ _____ _____</p> <p>If Yes: Starting vertex is: _____ Ending vertex is: _____</p>	<p>b)</p>  <p>Euler path exist? YES / NO (circle) Why: _____ _____ _____</p> <p>If Yes: Starting vertex is: _____ Ending vertex is: _____</p>
<p>c)</p>  <p style="text-align: center;">→</p>	<p>Euler path exist? YES / NO (circle) Why: _____ _____ _____</p> <p>If Yes: Starting vertex is: _____ Ending vertex is: _____</p>

8. Identify a simple circuit of length 3 in each one of the graphs a), b) and c) of question 1.

Graph a) _____

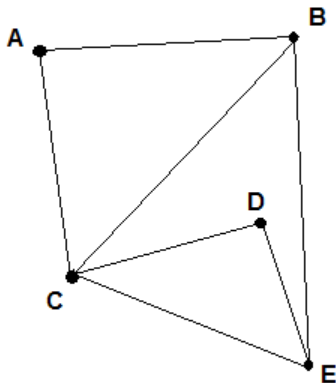
Graph b) _____

Graph c) _____

What is the distance $d(M, S)$? _____

9. Determine if there is a Hamiltonian circuit in each of the graphs below.

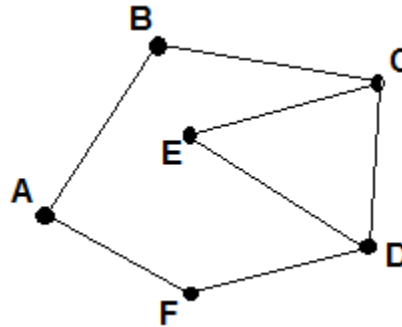
a)



Does a Hamiltonian circuit exist? YES / NO

If Yes write the path:

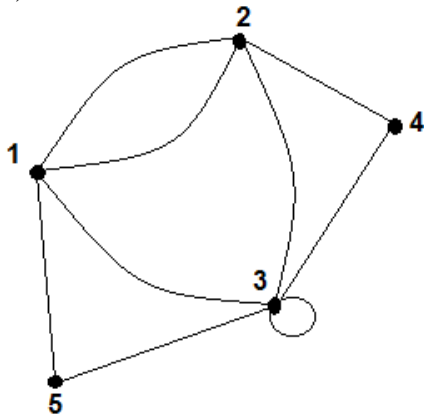
b)



Does a Hamiltonian circuit exist? YES / NO

If Yes write the path:

c)

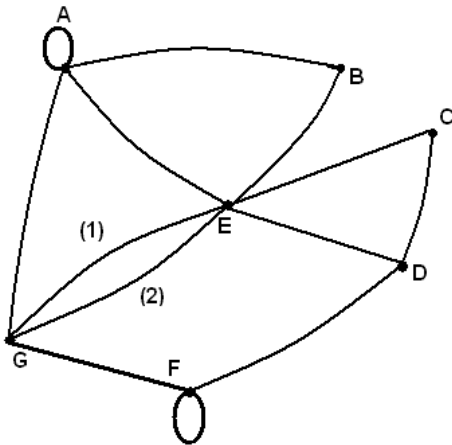


Identify how many edges must be added to the graph in b) so you can have a complete graph

Does a Hamiltonian circuit exist? YES / NO

If Yes write the path:

10. A relation is defined by the graph G as shown below:

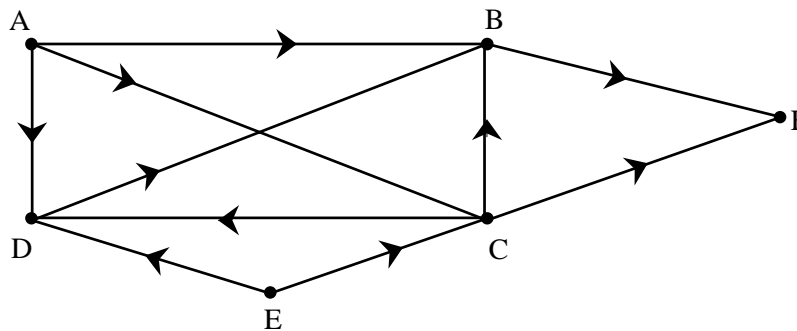


- a) Determine the sum of the degrees of the adjacent graph.
- b) Without listing the edges extensively or tracing the path over the graph, which vertex would you start from and which vertex would you end at to be able to walk on every edge once (including the loops)?

STARTING VERTEX: _____

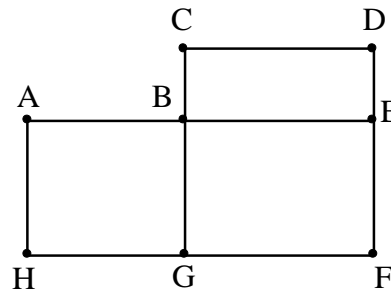
ENDING VERTEX: _____

11. Given the following graph.



- 1) Which of the following statements is true?
 - A) There is a path that begins at vertex B and ends at vertex D.
 - B) There is a path that begins at vertex C and ends at vertex A.
 - C) There is a path that begins at vertex D and ends at vertex C.
 - D) There is a path that begins at vertex E and ends at vertex B.
- 2) Which edge can you switch direction so that you can have the longest circuit? Determine its length.

12. The edges of the graph on the right represent the streets in a residential area.



While on patrol, a police officer takes different routes.

Indicate whether each of the following statements is true or false (T or F):

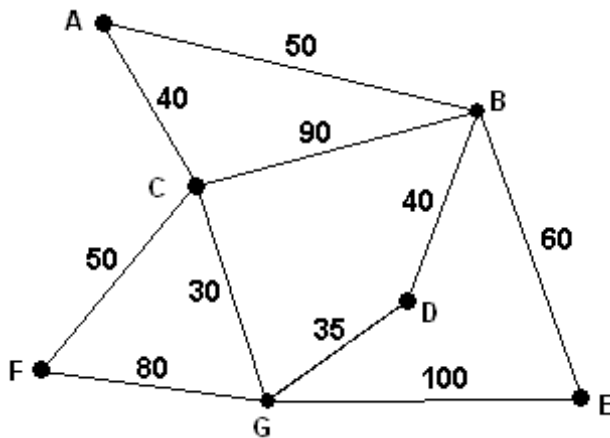
_____ this graph contains a Euler path. *Explain using the rule:* _____

_____ this graph contains a Euler circuit. *Explain using the rule:* _____

_____ this graph contains a Hamiltonian chain.

_____ this graph contains a Hamiltonian cycle.

13. Consider the graph below:

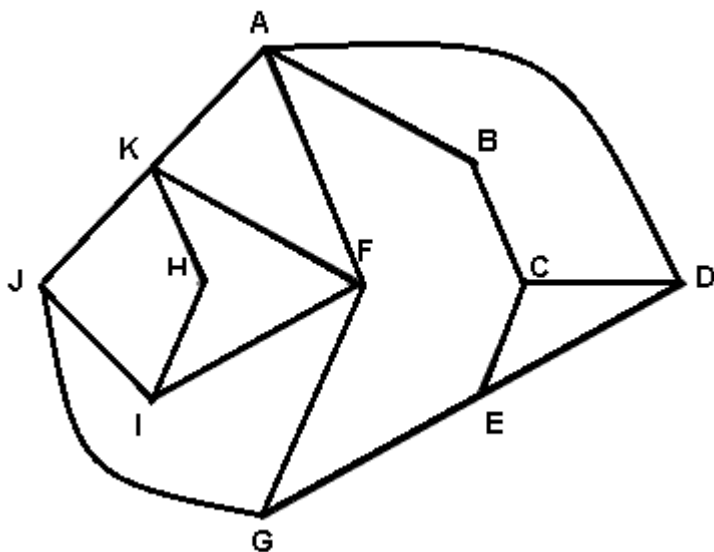


a) Determine $d(A, E)$: _____

b) Determine the tree of minimum value by highlighting it over the graph on the left and determine its total weight value.

Total weight value of the tree is: _____

14. In the graph below, identify two edges that to be removed in order to obtain an Euler path.

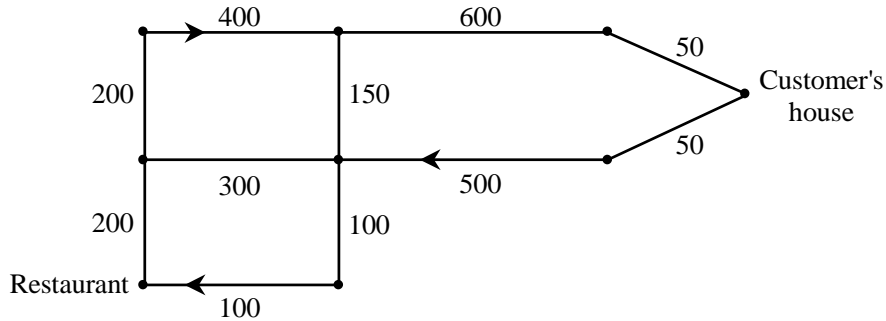


The two edges that can be removed in order to obtain an Euler path are: _____ and _____

Part B: Path of Minimum/Maximum Value

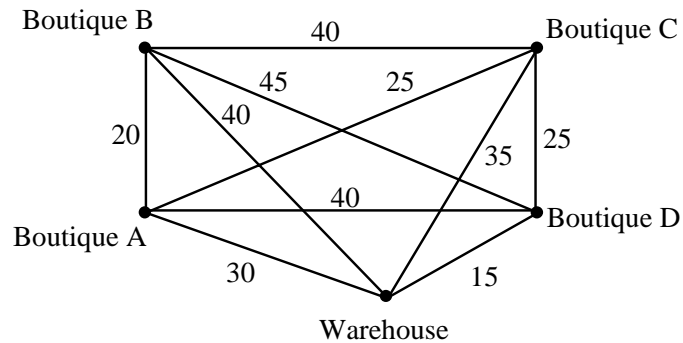
15. Ken works as a deliveryman for a restaurant. He must leave the restaurant, deliver a meal to a customer's house and return to the restaurant.

In the following graph, the edges represent the different streets that Ken can take. Some are one-way streets. The vertices of the graph represent the various intersections. The number on each edge represents the distance, in metres, between two intersections.



What is the shortest distance that Ken can travel to make this delivery?

16. The edges of the following graph represent the different routes Louise can take. The number on each edge indicates the time in minutes needed to get from one place to another.



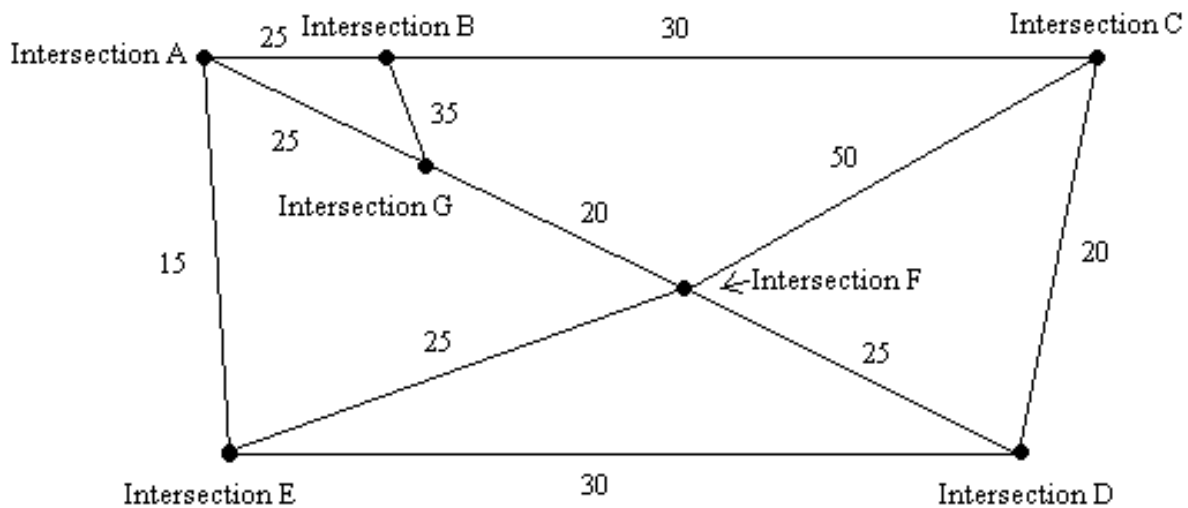
In choosing her route, Louise must consider the following constraints:

- Her route must begin and end at the warehouse.
- She is required to visit each of the four boutiques only once.
- She must go to Boutiques B and C before going to Boutique A.
- She must go to Boutique C before going to Boutique D.
- She wants to minimize the time it takes to make all her deliveries.

Which route should Louise take?

17. The edges of the graph below represent the routes that a mobile canteen can take. The values correspond to the revenue generated by taking that particular route

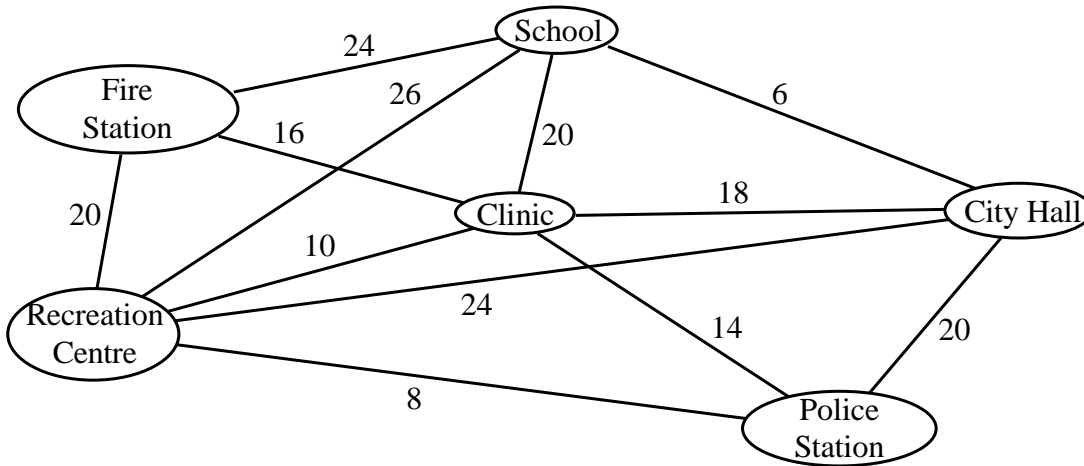
Routes that a mobile canteen can take



- a) Last week, the canteen’s route started at Intersection **D** and crossed each intersection only once. What was the route that generated the maximum revenue?
- b) This week, two routes will be removed out of the canteen’s itinerary so that it will be able to visit every route once. Identify these two routes that can be removed while still targeting maximum revenue. Also identify which intersections the canteen will have to start from and end at.
- c) Compare the difference in revenue from last week and this week’s revenues.

Part C: Tree of Minimum / Maximum Value

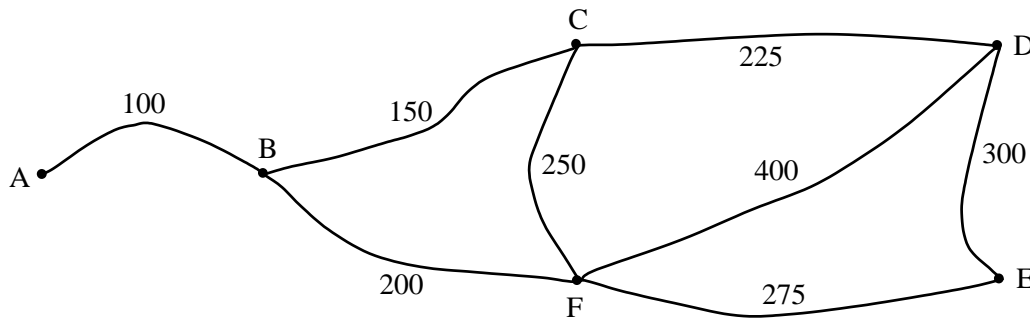
18. A piping system must be installed to supply water to public buildings in a city. Each edge in the following graph represents a possible section of this piping system. The number of each edge indicates the installation cost in thousands of dollars for that section of the system.



What is the minimum cost of installing this system while ensuring that all locations are somewhat connected indirectly?

- A) \$54 000
- B) \$58 000
- C) \$72 000
- D) \$88 000

19. A set of snowmobile trails connects villages A, B, C, D, E and F. In the graph below, the edges represent the trails. The number on each edge represents the number of snowmobilers who normally use that trail.

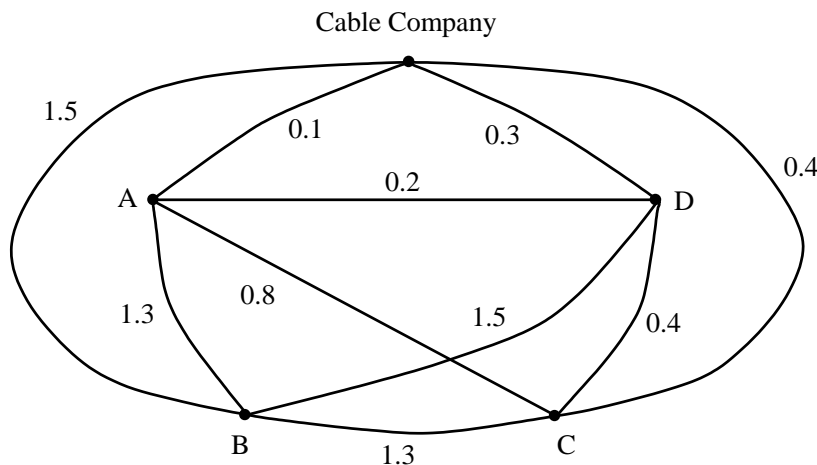


Some trails are closing to limit the amount of maintenance work required. When choosing to close some trails and to keep others open, the authorities must:

- ensure that each village can be reached
- keep the busiest trails open, if possible
- close as many trails as possible.

What is the graph representing the trails that remain open?

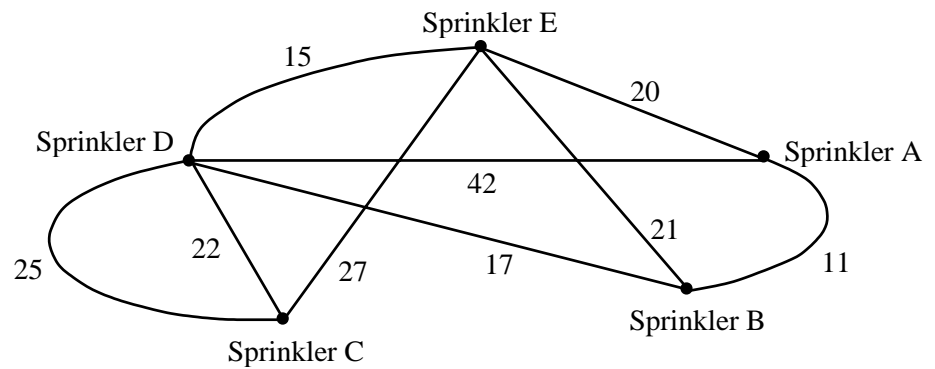
20. A cable TV company wants to connect cities A, B, C and D to its distribution network. In the following graph, the number on each edge indicates the installation cost (in millions of dollars) for each possible section of this network.



What is the minimum cost of connecting cities A, B, C and D to this network?

21. Steve wants to connect all the sprinklers so as to minimize the total combined length of the pipes.

What is the shortest length of piping required to connect all the sprinklers?



Part D: Critical Path Problems

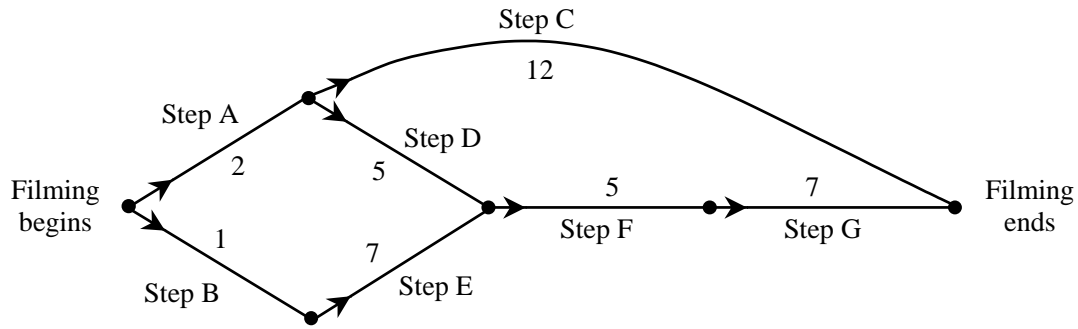
Very important note!!!! : The *minimum time* required to complete all tasks/steps involved in a critical path problem is the path with the *longest time*. If that path is not taken as the critical path then other tasks may be missed or are not completed.

22. The following table lists the tasks involved in producing an end-of-year examination.

Tasks	Time Required	Prerequisite Task(s)
A. Writing the French-language version of the exam	12 days	none
B. Translating the exam into English	3 days	A
C. Initial linguistic revision of the French version	2 days	A
D. Having the exam validated by a committee of teachers	2 days	B and C
E. Final linguistic revision of the French version	2 days	D
F. Final linguistic revision of the English version	1 day	D
G. Printing the exam and sending it to the schools	10 days	E and F

How many days are required to complete all these tasks?

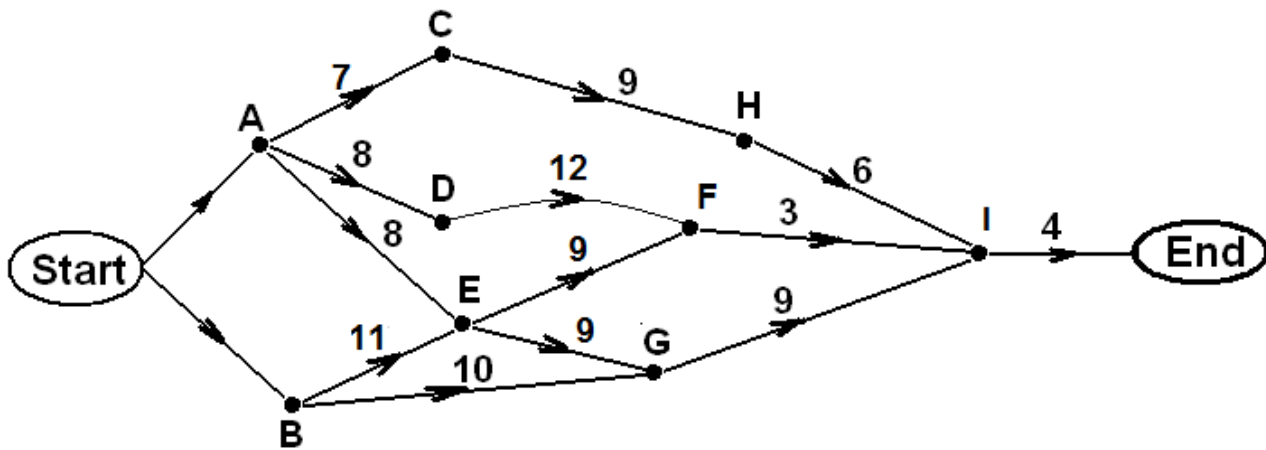
23. The following graph represents the different steps involved in filming a movie. Several steps can be carried out at the same time. The number on each edge indicates the number of days needed to complete the corresponding step.



It costs \$5000 per day to film the movie. After reviewing the situation, the producers realize that it will take 3 more days to complete step D. The total minimum cost of filming the movie will therefore increase.

By how much will the total minimum cost of filming the movie increase?

24. The steps to complete a given process are represented by the diagram below. The weight on every edge represents the time in days needed to carry out a step. The steps are represented by the letters followed by the duration edges.



- a) Determine the minimum time to complete the entire process.
- b) Because of a new constraint the time to complete **step B** will be reduced to 7 days. Does this new constraint affect the time to complete the whole process? Justify your answer.

Show detailed work

25. The following table lists these tasks, the time required to complete each, and the task(s) that must be carried out before performing each task.

Tasks		Time Required (weeks)	Preceding Task
A	Writing the text	1	none
B	Typing the text on the computer	2	A
C	Taking photographs	3	none
D	Editing the text	2	B
E	Typesetting the text and photographs	2	C and D
F	Selling advertising space	8	none
G	Typesetting the advertisements	2	F
H	Designing the cover page	1	none
I	Assembling the pages of the yearbook	1	E, G and H
J	Printing the yearbook	6	I

The yearbook production committee has decided that the time allotted to selling advertising space will be reduced to 4 weeks.

How will this reduction affect the total time required to produce the yearbook?

Show all your work.

26. A caterer must deliver a buffet by the end of the day. The following table shows the different steps involved in preparing this buffet. The caterer can complete several steps at the same time.

Step	Time (min)	Prerequisite Task(s)
A Cooking the meat	45	None
B Slicing the bread	10	None
C Washing the vegetables	25	None
D Preparing the sandwich spreads	50	A
E Making the sandwiches	65	B and D
F Cutting up the vegetables	15	C
G Preparing the dip	20	F
H Packaging the buffet	10	E and G
I Loading the buffet into the truck	5	H
J Delivering the buffet	35	I

- a) What is the minimum time required to complete the whole process of preparing the buffet?
- b) The customer who ordered this buffet called the caterer to ask him to add a large mushroom pizza. The mushrooms are already pre-sliced but they must be washed first. This dish takes 75 minutes to make. The pizza must be ready before the buffet can be packaged. Will this extra dish change the minimum time required to prepare the buffet?

27. Fancy Feast

Preparing for an elaborate meal involves many different steps. Some steps can be carried out simultaneously while others cannot be performed until one or more of the preceding tasks have been completed.

The following table lists the tasks, the time required to complete each task, and the task(s) that must be carried out before performing each task.

Task		Time Required (in minutes)	Prerequisite Task(s)
A	Deciding on the menu	15	-
B	Shopping for ingredients	40	A
C	Marinating the chicken	40	B
D	Washing the broccoli	3	B
E	Preparing the salad	15	B
F	Broiling the chicken	25	C
G	Steaming the broccoli	7	D
H	Washing & scrubbing the potatoes	10	E
I	Baking the potatoes	20	H

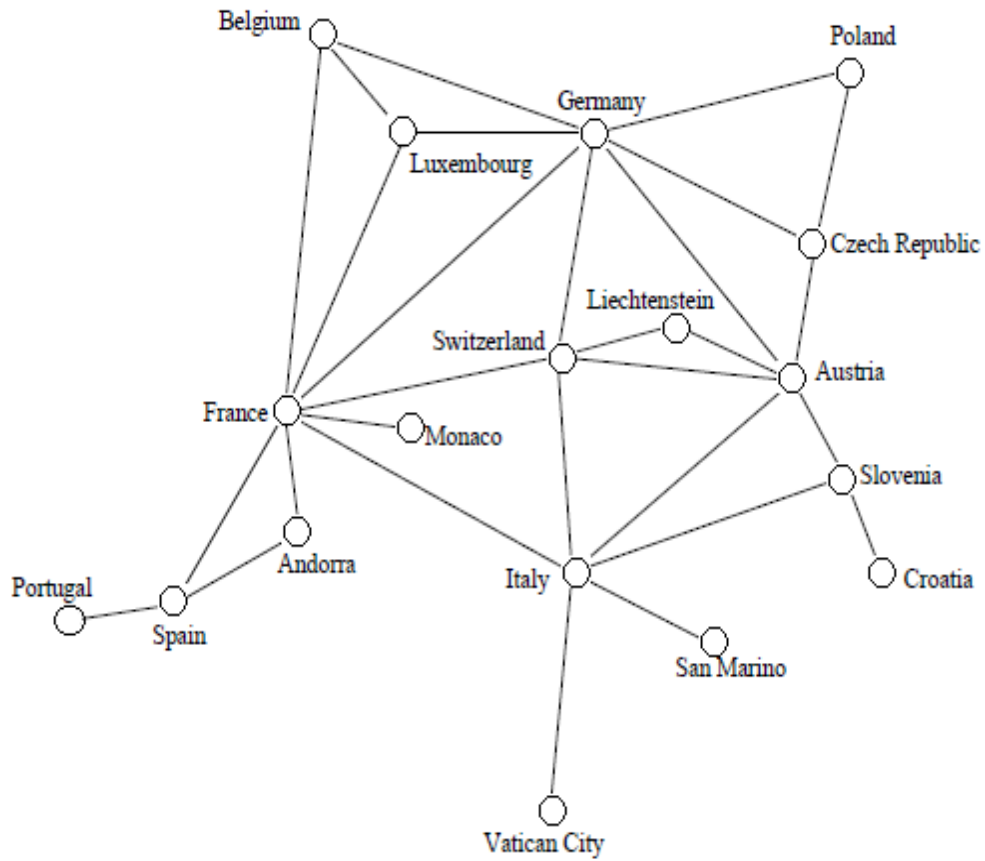
After flipping through a magazine you come across a recipe for “Twice Baked Potatoes” that you would like to include in your meal. This means that you must now add an additional step to complete your meal:

Task		Time Required (in minutes)	Prerequisite Task
J	Mashing and baking the potatoes again	30	I

How will this change in menu effect the total time required to prepare the meal?

Show all your work.

29.



What is the chromatic number of the graph above?

30. Nine prisoners must be transported from a detention centre to a courtroom for a trial. Some of them are police informers giving testimony; others are sworn enemies. As a result, certain prisoners cannot ride together in the same police van. For security reasons, prisoners have been labelled with letters as show in the relation table below.

A	Cannot ride with	B, C, D, and G
B	Cannot ride with	A, C, and E
C	Cannot ride with	A, B, and D
D	Cannot ride with	A and C
E	Cannot ride with	B and F
F	Cannot ride with	E
G	Cannot ride with	A
H	Cannot ride with	F and G
I	Cannot ride with	E and F

- a) Construct a graph that represents this situation.
- b) Determine the number of police vans needed to transport these prisoners to court.

31. A manager prepares work schedules for her nine employees. In the table below, an X signifies that the two employees cannot work simultaneously.

	Empl. A	Empl. B	Empl. C	Empl. D	Empl. E	Empl. F	Empl. G	Empl. H	Empl. I
Employee A		X		X			X	X	
Employee B	X			X		X		X	
Employee C							X		X
Employee D	X	X							X
Employee E									
Employee F		X					X		X
Employee G	X		X			X			
Employee H	X	X							
Employee I			X	X		X			

- Is it possible to create teams of three employees if each employee can only be a part of one work team?
- The manager must find someone to replace her if she is absent. Which employee should she identify? Explain.

SECTION 3: Optimization

Part 1: Multiple-choice questions. Each question is worth 4 marks.

1. Ms. Bouchard wants to minimize the renovation cost of her home. This includes plumbing and electrical jobs. The time required for the electrical work is more than 72 hours and does not exceed 96 hours. The time needed for plumbing must be at most twice as much as for the electrical work. In addition, a maximum of 168 hours are allotted to the renovation. The plumbing job costs \$42/h and the electrical work, \$54/h.

x: the time allotted in hours for the plumbing job

y: the time allotted in hours for the electrical job

Which of the following describes this situation?

A) Optimizing rule: $\text{Cost}(\$) = 42x + 54y$

Constraints:

$$\begin{aligned}x &\geq 0 \\y &\geq 0 \\y &> 72 \\y &\leq 96 \\2x &\leq y \\x + y &\leq 168\end{aligned}$$

C) Optimizing rule: $\text{Cost}(\$) = 42x + 54y$

Constraints: :

$$\begin{aligned}x &\geq 0 \\y &\geq 0 \\y &> 72 \\y &\leq 96 \\x &\leq 2y \\x + y &\leq 168\end{aligned}$$

B) Optimizing rule: $\text{Cost}(\$) = 42x + 54y$

Constraints: :

$$\begin{aligned}x &\geq 0 \\y &\geq 0 \\y &> 72 \\y &\leq 96 \\x &\geq 2y \\x + y &\geq 168\end{aligned}$$

D) Optimizing rule: $\text{Cost} = 42x + 54y$

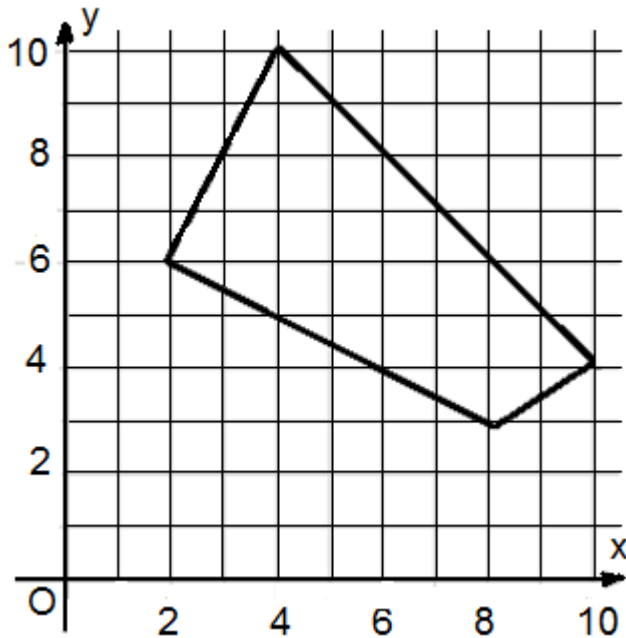
Constraints: :

$$\begin{aligned}x &\geq 0 \\y &\geq 0 \\y &> 72 \\y &\leq 96 \\x &\leq 2y \\x + y &\geq 168\end{aligned}$$

0	4
---	---

2. A future housing development will consist of duplexes (houses with two apartments) and quadruplexes (houses with four apartments).

The promoters of this development would like to minimize the building cost of the apartments. The following polygon represents the different possible combinations of duplexes and quadruplexes that can be built.



The cost of building the apartments is determined by the following rule:

$$N = 120\,000x + 240\,000y$$

Where x : number of duplexes
 y : number of quadruplexes

How many possible solutions minimize the cost of building the apartments?

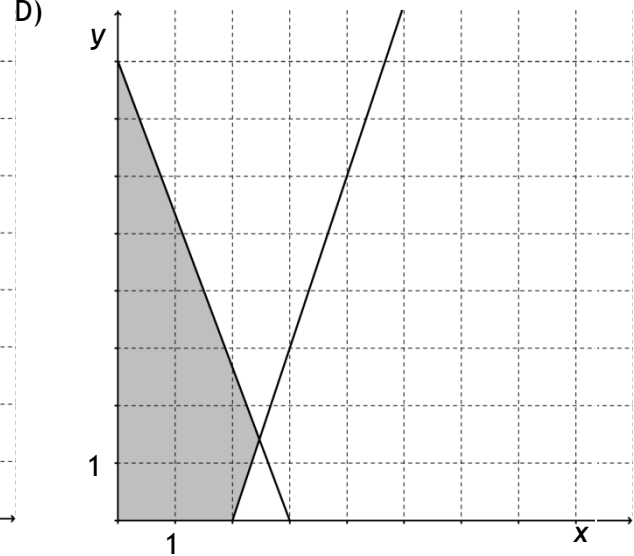
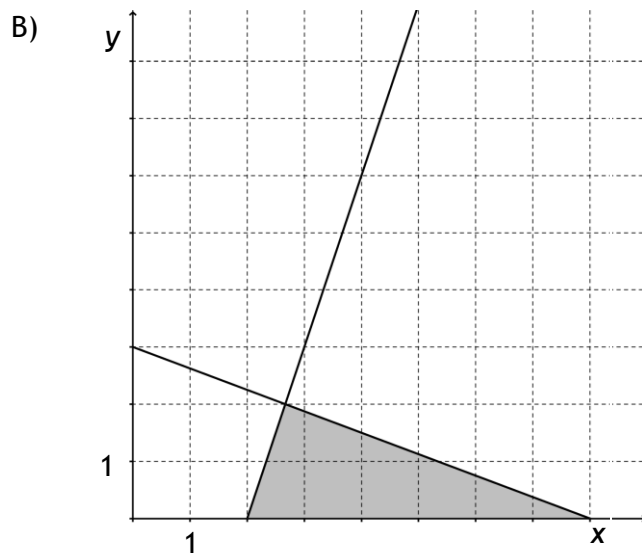
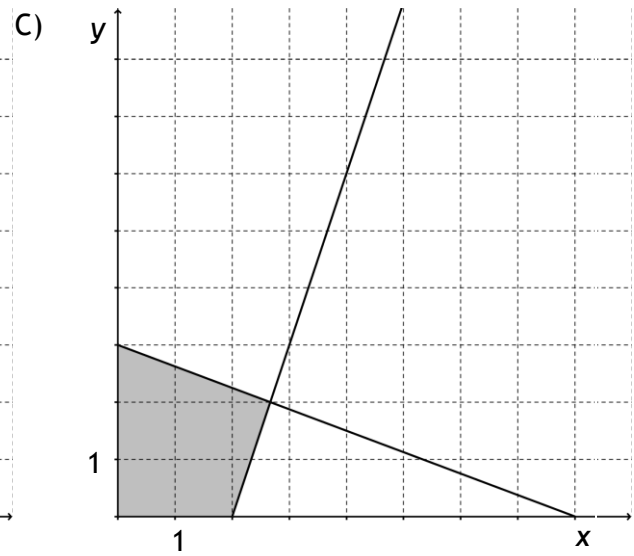
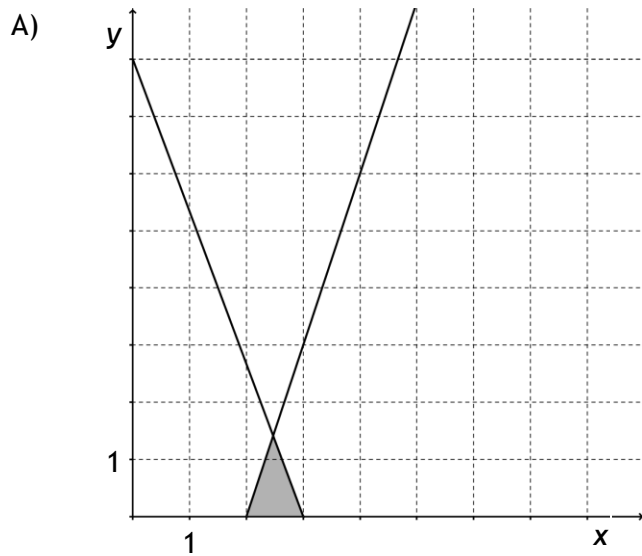
- A) 1
- B) 2
- C) 3
- D) 4

0	4
---	---

3. The constraints related to a situation are represented by the following system of inequalities:

$$\begin{aligned} x &\geq 0 \\ y &\geq 0 \\ 3x + 8y &\leq 24 \\ y + 6 &\leq 3x \end{aligned}$$

Which polygon of constraints represents this situation?



0	4
---	---

Part 2: Short answer. Each question is worth 4 marks.

4. Determine the coordinates of the vertex formed by the intersection of the boundary lines represented by the inequalities (5) and (6)

(1) $x \geq 0$

(2) $y \geq 0$

(3) $2y - 5 < 0$

(4) $x + y \geq 15$

(5) $-8x + 4y + 60 \leq 0$

(6) $y \geq 0.75x$

The coordinates of the intersection point formed by the boundary lines of the inequalities (5) and (6) are:

($x =$ _____ ; $y =$ _____)

0	2	4
---	---	---

5. The student council organized a dance. Near the end they recorded the following facts:

- Less than 450 tickets were sold
- Students from the school were at least twice as numerous as outsiders
- There were a minimum of 100 outside participants but no more than 150

x : the number of students

y : the number of outsiders

Write the constraints of this situation.

Constraints:

$$x \geq 0 \text{ and } y \geq 0$$

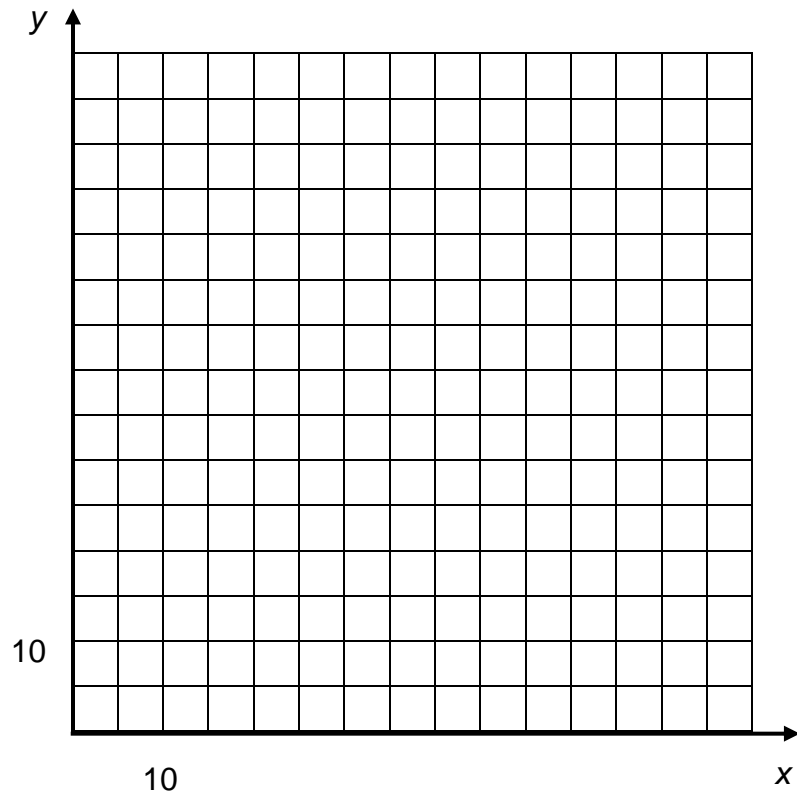
0	2	4
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Part 3: Long answer. Each question is worth 10 marks.

6. Cynthia is a member of the Young Achievers Club at her school. Her project requires her to paint a design on vases and sugar bowls. It takes her 2 hours to paint the design on a vase and 3 hours to do the same on a sugar bowl. During the school year, she can devote a maximum of 120 hours to her project and she expects to paint a maximum of 50 objects. In response to customer demand, she must paint at least 10 sugar bowls. Each painted vase will be sold for \$14 and each painted sugar bowl, \$10.

How many painted vases and sugar bowls must she sell to maximize her profit?

Let x : the number of vases
 y : the number of sugar bowls



(Four squares = 10 x 10 units)

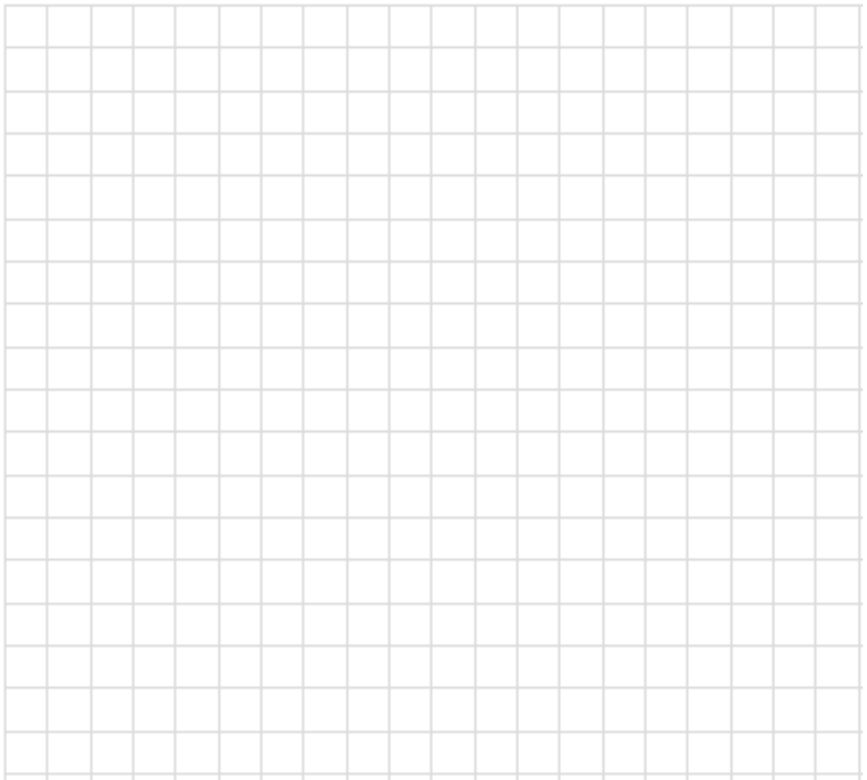
Answer: To maximize her profit, Cynthia must sell _____ vases and _____ sugar bowls

Use Mathematical Reasoning						
Cr1	40	32	24	16	8	0
Cr2	40	32	24	16	8	0
Cr3	20	16	12	8	4	0
TOTAL:						/100

7. Students at Kennedy High are participating in a Dress down Day to raise money for their favourite charity. Students in Secondary 1 and 2 will donate \$1.00, while students in Secondary 3, 4 and 5 will have to contribute \$2.00. No more than 800 students attend the school and no less than 700 will be present that day. The number of Secondary 1 and Secondary 2 students is greater than or equal to the number of Secondary 3, 4 and 5 students.

<p>Let x: number of students in Secondary 1 and 2 y: number of students in Secondary 3, 4 and 5</p>	<p>The constraints of the situation are represented by the following inequalities: $x \geq 0$ $y \geq 0$ $x + y \leq 800$ $x + y \geq 700$ $x \geq y$</p>
--	---

What is the maximum amount of money the school can expect to raise?



Answer: The maximum amount of money the school can expect to raise is _____

	Use Mathematical Reasoning							
Cr1	40	32	24	16	8	0		
Cr2	40	32	24	16	8	0		
Cr3	20	16	12	8	4	0		
	TOTAL:						/100	

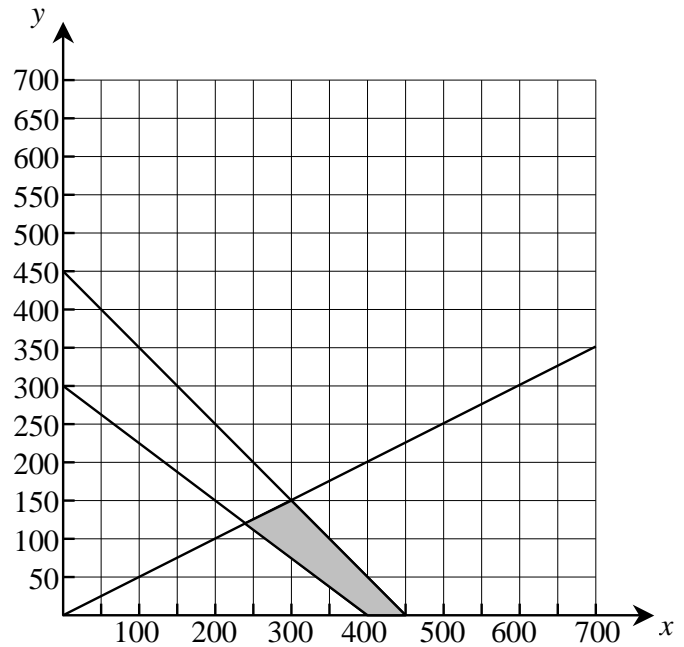
8. Tickets are on sale for the presentation of *Cheaper by the Dozen* at Laurentian Regional High School. Tickets cost \$15 per student and \$20 per adult. The auditorium can seat 450 people. The school hopes to sell at least \$6000 worth of tickets. The number of student tickets sold is always greater than or equal to twice the number of adult tickets sold.

The constraints of this situation are:

$$\begin{aligned} x &\geq 0 \\ y &\geq 0 \\ x + y &\leq 450 \\ 15x + 20y &\geq 6000 \\ x &\geq 2y \end{aligned}$$

where x is the number of student tickets sold,
 y is the number of adult tickets sold.

The polygon of constraints of this situation is shown below.



The school makes a profit of \$10 per student ticket sold and \$15 per adult ticket sold.

Because tickets sales have been much better than expected, extra seats have been added, bringing the total number up to 600.

By how much will the school’s maximum profit increase because of the change in ticket sales?

Answer: The school's maximum profit will increase by \$ _____

	Use Mathematical Reasoning					
Cr1	40	32	24	16	8	0
Cr2	40	32	24	16	8	0
Cr3	20	16	12	8	4	0
	TOTAL:					/100