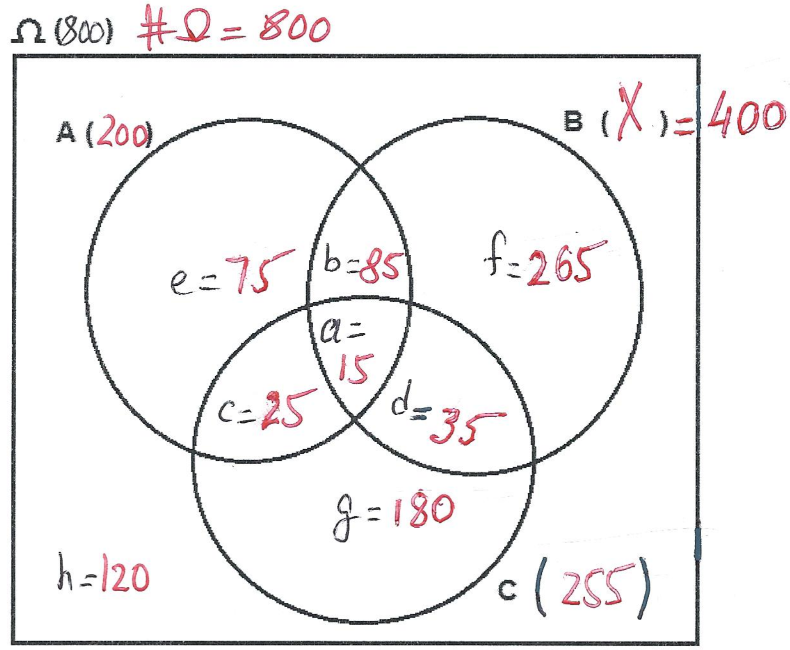
11. A, B, and C are events (or subsets) of a universal set Ω, where #Ω = 800



<u>Givens:</u>

a) #(A∩B∩C) = 15
b) P(A) = 1/4
c) P(A∩B) = 1/8
d) P(C|A) = 1/5
e) P[(A∩B)U(A∩C)U(B∩C)] = 20%
f) #(AUBUC)'=120
g) P(C|B) = 12.5%

<u>Question:</u> Complete the Venn diagram above and determine P (A|C).

<u>SOLUTION</u>:

a) $\#(A\cap B\cap C) = 15 \rightarrow a = 15$

b) $P(A) = \frac{1}{4} \rightarrow \#A = \frac{1}{4}$ of $800 = 200$

c) $P(A\cap B) = \frac{1}{8} \rightarrow \#A\cap B = \frac{1}{8}$ of $800 = 100$

$\rightarrow b = 100 - 15 = 85$

d) $P(C|A) = \frac{1}{5} \rightarrow \frac{c+a}{200} = \frac{1}{5}$

$\rightarrow c + a = \frac{200}{5}$

$\rightarrow c + 15 = 40$

$\rightarrow c = 25$

e) $P((A\cap B)\cup(A\cap C)\cup(B\cap C)) = 20\%$

$\rightarrow \#((A\cap B)\cup(A\cap C)\cup(B\cap C)) = 20\%$ of 800

$\rightarrow$  $= 20\%$ of $800 = 160$

$\rightarrow d = 160 - (a + b + d)$

$\rightarrow d = 160 - (15 + 85 + 25)$

$d = 35$

$e = 200 - (25 + 15 + 85) = 75$

f) $\#(A\cup B\cup C)' = 120$

$\rightarrow h = 120$

g) $P(C|B) = 12.5\% = \frac{a+d}{X}$

$X = \#B$

$\rightarrow 0.125 = \frac{15+25}{X}$

$\rightarrow X = \frac{50}{0.125}$

$\rightarrow X = 400 \rightarrow \#B = 400$

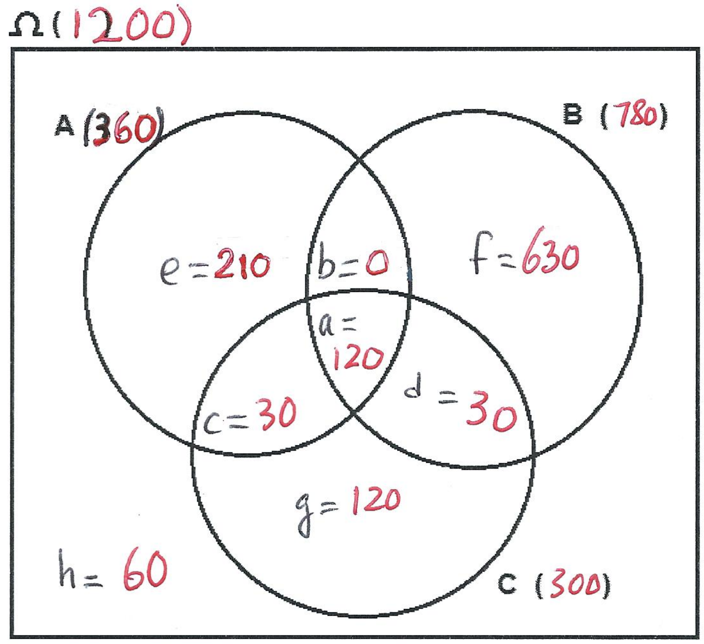
$\rightarrow f = 400 - (85 + 15 + 35) = 265$

$\rightarrow g = 800 - (120 + 65 + 85 + 265 + 35 + 15 + 25)$

$\rightarrow g = 180 \rightarrow \#C = 25 + 15 + 35 + 180 = 255$

$\therefore \boxed{P(A|C) = \frac{15+25}{255} = \frac{40}{255}}$

12. A, B, and C are events (or subsets) of a universal set Ω, where #Ω = 1200



Givens:

a) P(A∩B∩C) = 10%
b) P(B∩C) = 1/8
c) P(C) = 25%
d) P[(AUBUC)']=5%
e) P(A|C) = 0.5
f) P(A) = 30%
g) P(B|A)=1/3

Question:

a) Complete the Venn diagram above.

b) Rachel claims that at least 50% of Ω do not have anything in common with A and C and that these 50% are only in B. Is she right?

c) Determine P (C|B).

ANSWER:

$\rightarrow \#\Omega = 1200$

a) $P(A\cap B\cap C) = 10\%$

$\rightarrow \#A\cap B\cap C = 10\%$ of $1200 = 120$

$\rightarrow a = 120$

b) $P(B\cap C) = \frac{1}{8} \rightarrow \#B\cap C = \frac{1}{8}$ of 1200

$\rightarrow \#B\cap C = 150$

$\rightarrow d = 150 - 120 = 30$

c) $P(C) = 25\% \rightarrow \#C = 0.25\,(1200)$

$\rightarrow \#C = 300$

d) $P[(A\cup B\cup C)'] = 5\%$

$\rightarrow \#(A\cup B\cup B)' = 0.05 \times 1200 = h$

$\rightarrow h = 60$

e) $P(A|C) = 0.5$

$\rightarrow \frac{120+c}{300} = 0.5$

$\rightarrow 120 + c = 150$

$\rightarrow c = 30$

$\rightarrow P(A) = 30\% \rightarrow \#A = 0.3 \times 1200 = 360$

$\rightarrow e = 360 - (30 + 120 + 30) = 180$

g) $P(B|A) = \frac{1}{3} \rightarrow \frac{120+b}{360} = \frac{1}{3}$

$\rightarrow 3(120+b) = 360(1) \rightarrow 360 + b = 360$

$\rightarrow b = 0$

$\rightarrow e = 360 - (30 + 120 + 0) = 210$

$g = 300 - (30 + 120 + 30) = 120$

$\rightarrow f = 1200 - (60 + 120 + 30 + 120 + 30 + 210 + 0) = 630.$

∴ Rachel is right, more than 50% are in B only. $\rightarrow \#B = 0 + 120 + 30 + 630 = 780$

c) $P(C|B) = \frac{120+30}{780} \rightarrow \boxed{P(C|B) = \frac{150}{780}}$

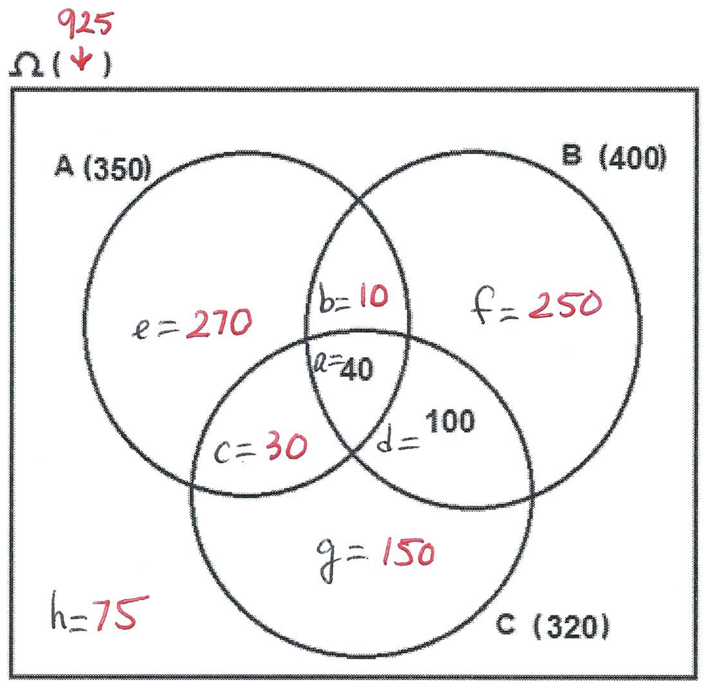
13. A, B, and C are events (or subsets) of a universal set Ω.

Givens:

1) $\#(A \cup B \cup C)' = 75$
2) $P(A|B) = 1/8$
3) $P(C|A) = 20\%$

Questions:

a) Complete the Venn diagram on the right.
b) Determine #(Ω)
c) Determine P (C*), where C* represents a set of values that belong exclusively or strictly to C only.



ANSWER:

a)

1) $\#(A \cup B \cup C)' = 75 \rightarrow h = 75$

2) $P(A|B) = \frac{1}{8} = \frac{b+40}{400}$

$\rightarrow 400 = 8b + 8(40)$

$400 = 8b + 320$

$80 = 8b$

$\rightarrow 10 = b$

3) $P(C|A) = 20\%$ or 0.20

$\rightarrow 0.20 = \frac{c+40}{350}$

$\rightarrow 0.20(350) = c + 40$

$70 = c + 40$

$\rightarrow c = 30$

$\rightarrow e = 350 - (30 + 40 + 10) = 270$

$g = 320 - (30 + 40 + 100) = 150$

$\rightarrow f = 400 - (10 + 40 + 100) = 250$

b) $\#\Omega = 75 + 150 + 30 + 40 + 100 + 270 + 10 + 250$

$\rightarrow \#\Omega = 925$

c) $\boxed{P(C^*) = \frac{150}{925}}$

14. A survey of 1200 students was conducted at this local High School about their involvement in three particular team sports, soccer, basketball and rugby. The survey shows that:

- The probability of selecting at random a student who plays all three sports is 1 / 10
- 80 do not play any of the three-team sports
- 500 play soccer
- 1/4 play rugby
- 220 play soccer and basketball
- The probability of selecting a student who plays soccer and rugby is 1 / 6
- The probability of selecting a student who plays basketball given that he or she plays rugby is 60%.

Question:
Draw a Venn diagram for this situation and determine the probability of finding a student who plays rugby given that he/she plays basketball.

ANSWER → Quiz SOLUTIONS (question # 13) → online

16. A school organized a ski trip for Secondary V students. The following table shows the distribution of students on that trip.

Solution

	Girl	Boy	Total
Went snowboarding	205	(188)	393
Went downhill skiing	36	41	77
Went cross-country skiing	17	11	28
Total	258	240	498

A girl was selected at random from among the students who went on the trip.

a) What is the probability of selecting a girl who went downhill skiing?

b) Given that the student went snowboarding, what is the probability that it is a boy?

a) $P(\text{Girl} \cap \text{"Went downhill skiing"}) = \frac{36}{498}$

b) $P(\text{Boy} \mid \text{"Snowboarding"}) = \frac{188}{393}$

17. A study of speeding violations and drivers who use car phones produced the following data. The total number of people in the sample is 755.

SOLUTION

	Speeding violation in the last year (V)	No speeding violation in the last year (W)	Total
Car phone user (R)	$a = 25$	$b = 280$	$c = 305$
Not a car phone user (Q)	$45 = d$	$e = 405$	$f = 450$
Total	$g = 70$	$685 = h$	$755 = i$

1) • The probability of finding a person had a speeding violation last year given that he/she was not a car phone user is 10%

2) • The probability of finding a person who was a car phone user given that he/she had a speeding violation last year is 5/14

Using the information above, fill the table and calculate the following probabilities

a) P(person is a car phone user)
b) P(person had no violation in the last year)
c) P(person had no violation in the last year and was a car phone user)
d) P(person is a car phone user or person had no violation in the last year)
e) P (person had no violation last year given that person was not a car phone user)

ANSWER: R : "A car phone user"
Q : "Not a car phone user"
V : "Had a speeding violation in the last year"
W : "Had no speeding violation in the last year"

$g = 755 - 685 = 70$

$a = 70 - 45 = 25$ or use $P(R|V) = \frac{5}{14} = \frac{a}{70}$

2) $\rightarrow \frac{5(70)}{14} = a \rightarrow a = 25$

1) $P(V|Q) = 10\% = 0.1 = \frac{45}{f}$

$\rightarrow f = \frac{45}{0.1} = 450$

$\rightarrow e = 450 - 45 = 405$

$\rightarrow b = 685 - 405 = 280$

$\rightarrow c = 25 + 280 = 305$

a) $P(R) = \frac{305}{755}$

b) $P(W) = \frac{685}{755}$

c) $P(W \cap R) = \frac{280}{755}$

d) $P(R \cup W) = P(R) + P(W) - P(R \cap W)$
$= \frac{305}{755} + \frac{685}{755} - \frac{280}{755} = \frac{710}{755}$

e) $P(W|Q) = \frac{405}{450}$

18. SAVE A BUNDLE

Smart Media is a new company offering Home Phone, High-Speed Internet and Digital Cable TV services. To attract more teachers to switch to their company, they are offering special prices for educators:

Educator Specials	
Home Phone	\$25/month
High-Speed Internet	\$30/month
Digital Cable TV	\$50 / month

Teachers who choose to bundle two services will receive a 10% discount, and those choosing to bundle all three services will receive a 20% discount.

Of the staff at D'Arcy McGee and Symmes schools:

ANSWER:

- 48 have subscribed to at least one of the services → $\#\Omega = 48$
- 30 have signed up for the Digital Cable TV service → $\#C = 30$
- 10 have signed up for only the High-Speed Internet service → $\#H$ only $= 10 = f$
- 20 have bundled two or more services → $\#[(H\cap I)\cup(I\cap C)\cup(H\cap C)] = 20$

\#  = 20

If one of the subscribed teachers is chosen at random:

a) • the probability that they subscribed to all 3 services is 1/6 → $P(H\cap I\cap C) = \frac{1}{6} \rightarrow \#H\cap I\cap C = \frac{1}{6}$ of $48 = 8 \rightarrow a = 8$

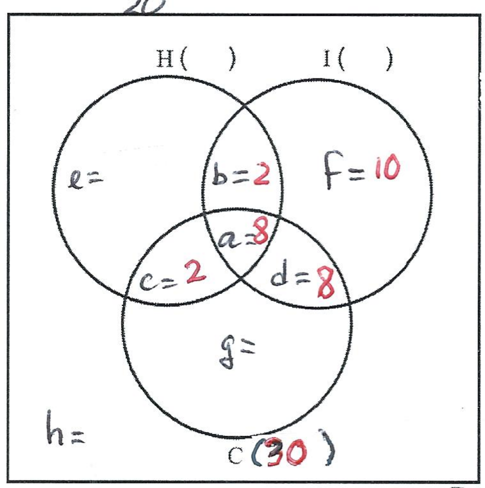
b) • the probability that a teacher subscribed to the Home Phone service, given that they signed up for the Digital Cable TV service is 1/3 → $P(H|C) = \frac{1}{3} = \frac{c+8}{30} \rightarrow 30 = 3c + 24 \rightarrow c = 2$

c) • the probability that they have subscribed to the Home Phone and High-Speed Internet services ONLY, given that they have bundled two or more services is 10% → $P($H & I only | "2 or more services"$) = 0.10$
$\rightarrow \frac{b}{20} = 0.10 \rightarrow b = 2$

Smart Media wants to encourage more teachers to bundle by showing them how much they could save with the bundle discounts.

What is the average monthly savings by a teacher who chooses to bundle two or more service?

H: subscribes to Home Phone
I: subscribes to High-Speed Internet
C: subscribes to Digital Cable TV



$\rightarrow d = 20 - (2+2+8) = 8$

Savings: H & I only: $2(25+30)\times 0.10 = \$11$
I & C only: $8(30+50)\times 0.10 = \$64$
H & C only: $2(25+50)\times 0.10 = \$15$
All three → $8(25+30+50)\times 0.20 = \$168$

Total savings of the 20 teachers
$168 + 15 + 64 + 11 = 258$
Average savings for 1 teacher:
$\frac{258}{20} = \$12.9$