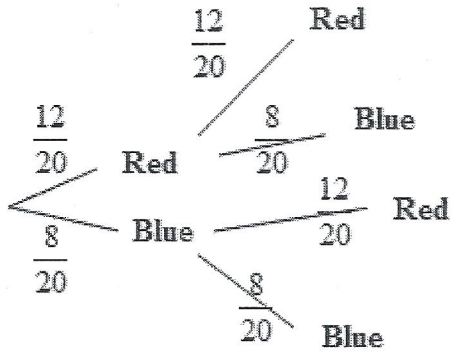**QUIZ: Probability (Part 1)**
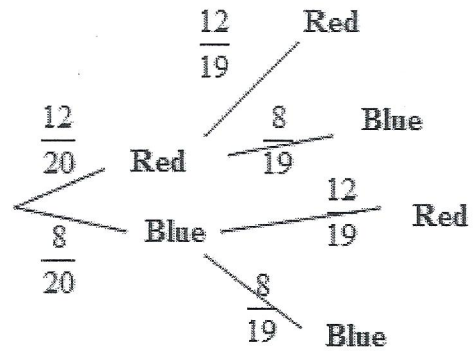January 19, 2018
Duration: 50 minutes

SOLUTIONS

74

1. A box contains 20 marbles, of which 12 are red and the rest are blue. Josephine randomly picks two marbles successively. Each time a marble is drawn it is kept outside the box. Which probability tree below correctly represents this situation? (4 marks)
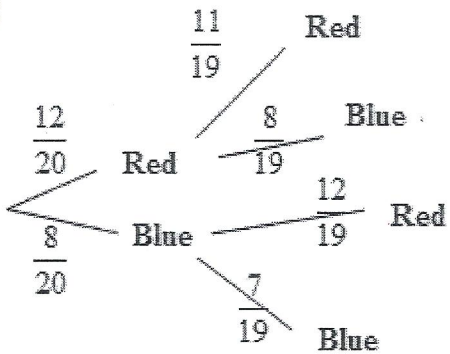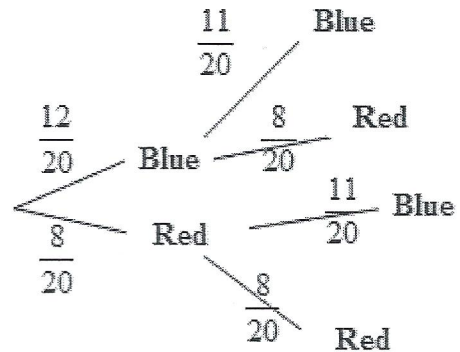
A)

C)

B) (circled)

D)

2. Suppose A and B are two events of a Universal set Ω, such that A∪B = Ω. (4 marks)

Which one of the scenarios below does not describe two independent events?

A)   A: Flipping a coin
     B: Flipping the same coin the second time

C)   A: Pick a blue marble and keep it out of the box
     B: Pick another blue another blue marble from the same box

B)   P(A)=4/9 and P(B)=5/9
     P(A∩B) = 4/9 x 5/9

D)   A: Roll a die and record the number shown
     B: Toss a coin and check if it's Heads or Tails

3. Which one of pair of events A and B below is not mutually exclusive? (4 marks)

A)   A: rolling a number less than 6 in a six-sided die
     B: rolling an even number in a six-sided die

C)   A: flipping a coin
     B: rolling a six-sided die

B)   A: drawing a blue marble from a box
     B: drawing a Queen of Hearts

D)   A: flipping a coin
     B: rolling a die

4. Based on the table seen below, Julie wants to know the probability of finding a student who speaks English given that student is a girl. (4 marks)

	English	Spanish	Total
Boy	20	15	35
Girl	45	25	70
Total	65	40	105

Which one of the answers below represents that probability?

A) $\frac{45}{105}$

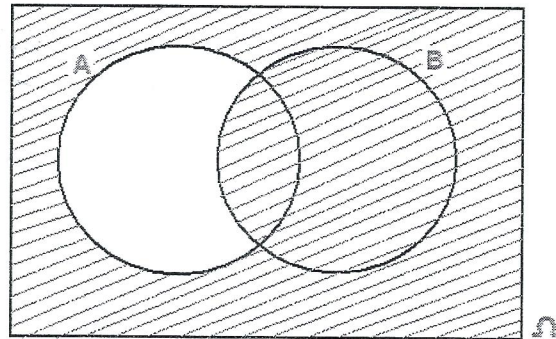C) $\frac{45}{70}$ ← P(English|Girl)

B) $\frac{70}{105}$

D) $\frac{45}{65}$

5. Which one of the builder notations below illustrates the shaded area in the diagram on the right? (4 marks)

1) (A U B)'

2) A' U B *(circled)*

3) A'∩B

4) A'∩B'

6. An experiment consists of flipping a coin then rolling a 6-sided die.

Calculate the probability of getting Tails followed by an even number: (4 marks)

$$P(\text{Tails} \cap \text{"Even Number"}) = \frac{1}{2} \times \frac{3}{6} = \frac{3}{12} \text{ or } \frac{1}{4} \text{ or } 25\%$$

$\uparrow$ implies multiplication

The probability of getting Tails followed by even number is $\underline{\frac{3}{12}}$ or $\frac{1}{4}$ or 25%

7. A and B are two events. Using the formulas below, fill in the missing values in the table: (4 marks)

Conditional Probability: $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

Probability of the union: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Event Probability	P(A)	P(A U B)	P(A | B)	P(B)	P(A∩B)
Value	0.40	0.82	0.125	0.48	0.06

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$
$$= \frac{0.06}{0.48} \rightarrow P(A|B) = 0.125$$
$$\text{or } 12.5\%$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 0.40 + 0.48 - 0.06$$
$$\rightarrow P(A \cup B) = 0.82$$
$$\text{or } 82\%$$

8. A box contains three red marbles and six blue marbles. An experiment consists of getting two consecutive marbles without placing the <u>first draw back into the box.</u>

Determine the probability of getting two blue marbles. (4 marks)

$$P(Blue \cap Blue) = \frac{6}{9} \times \frac{5}{8} = \frac{30}{72}$$

The probability of getting two blue marbles is ___ 30/72 or $\frac{15}{36}$ or $\frac{5}{12}$ or 0.41666... or 41.66...%
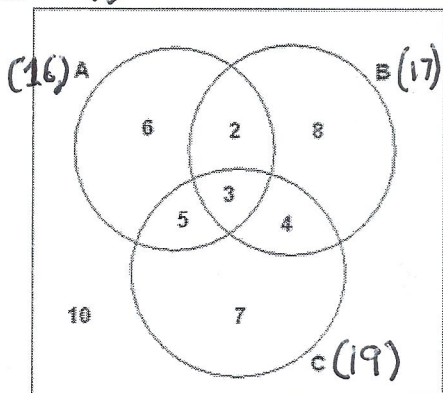
9. A, B and C represent three events or three groups of students.

A: Choosing a student who are left-handed
B: Choosing a student who wears glasses
C: Choosing a student who has brown hair

#Ω=45

Calculate the following probabilities: (6 marks)

a) P(B | A) = $\frac{5}{16}$ or 31.25%

b) P(B ∩ C) = $\frac{7}{45}$ or 15.55...%

c) P((A∪B∪C)') = $\frac{10}{45}$ or $\frac{2}{9}$ or 22.22...%

10. A Manufacturing company conducted a survey. The table below summarizes the results of the survey (6 marks)

	Part-Time	Full-Time	Total
Production (P)	25	18	43
Sales (S)	42	37	79
Warehouse (W)	168	143	311
Total	235	198	433

a) What is the probability of selecting an employee who works in the warehouse department?

$$P(W) = \frac{311}{433} \text{ or } 71.8\%$$

b) If a chosen employee works part-time, what is the probability that he/she works in the production department?

$$P(P | \text{"Part-Time"}) = \frac{25}{235} \text{ or } 10.6\%$$

c) Given that an employee works full-time, what is the probability that he/she works in the sales department?

$$P(Sales | \text{"Full-Time"}) = \frac{37}{198} \text{ or } 18.7\%$$

11. Unity Language School offers three courses Arabic, French, and Spanish. A student is enrolled to at least one course.

The Venn diagram below represents the distribution of 90 students according to courses they are currently taking.

Distribution of the students according to courses they registered in

Arabic(31)
(A)

French(40)
(F)

20   8   26

1

2   5

28

$\#\Omega = 90$

$\Omega(90)$

(S) Spanish (36)
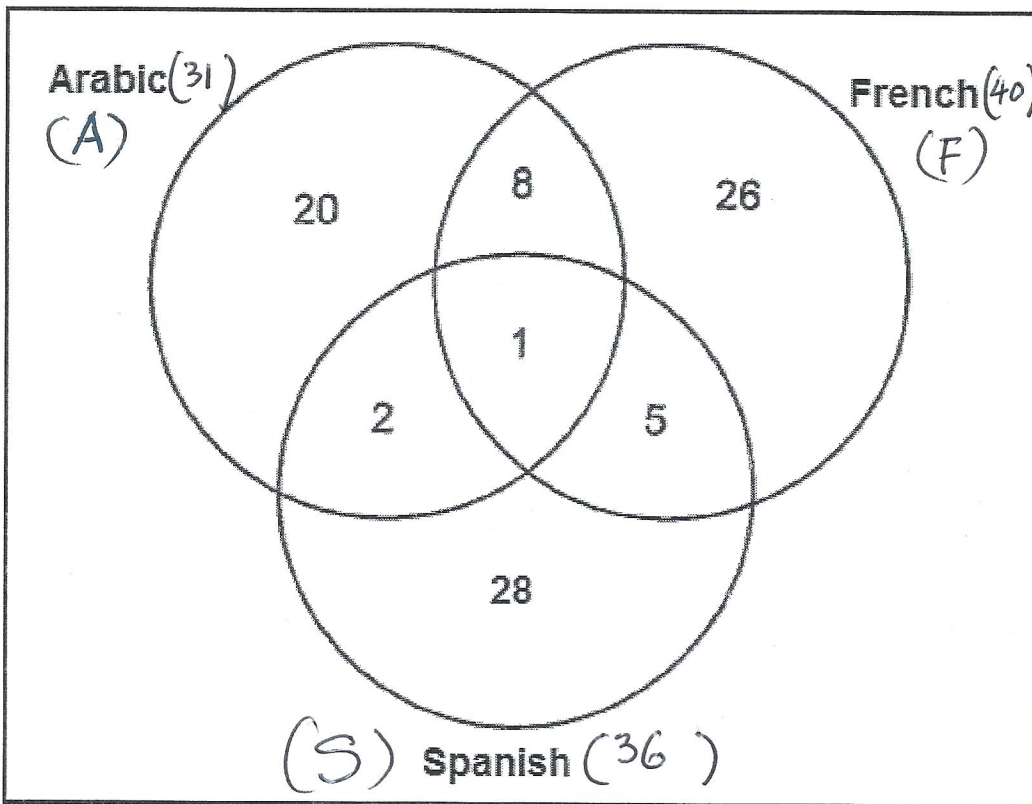
$\#\Omega = 20 + 8 + 26 + 2 + 1 + 5 + 28 = 90$

Calculate the following probabilities: (4 marks)

a) Given that the student chosen is studying French, what is the probability that he/she is studying Spanish?

$$P(S|F) = \frac{6}{40}$$

Answer: 6/40 or 3/20 or 15%

b) What is the probability of choosing a student who is studying Arabic or Spanish?

$$P(A \cup B) = (20 + 8 + 1 + 2 + 5 + 28)/90$$

Answer: 64/90 or $\frac{32}{45}$ or 71.11... %

12. Symmes School organized a field trip for the grade 7s and 8s. The following table shows the distribution of students on that trip.

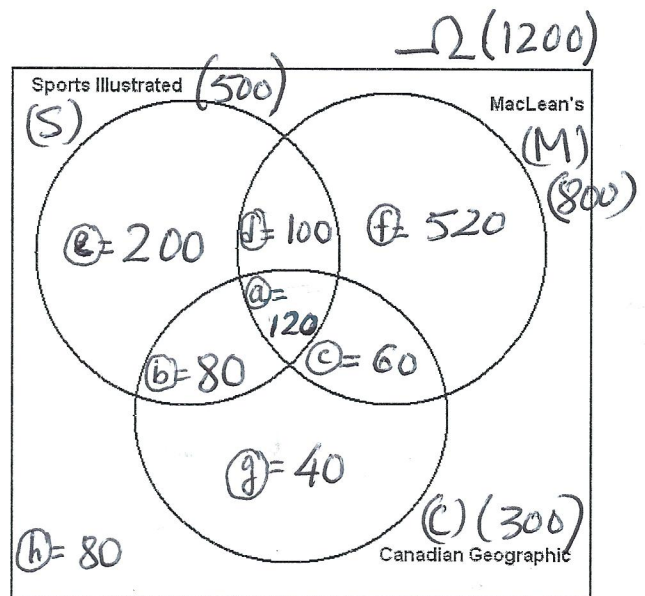|	Grade 7 student (G7)	Grade 8 student (G8)	Total
Went to Museum of Science & Technology (ST)	50	30	80
Went to Museum of Nature (N)	40	20	60
Went to Canadian Museum of History (H)	36	28	64
Total	126	78	204

Calculate the following probabilities: (6 marks)

a) P(G7 | ST) = $\frac{50}{80}$ or $\frac{5}{8}$ or 62.5%

b) P(N ∩ G8) = $\frac{20}{204}$ or $\frac{5}{51}$ or 9.8%

c) P(N | G8) = $\frac{20}{78}$ or $\frac{10}{39}$ or 25.6%

13. 1200 people were surveyed randomly to identify what news magazines people generally read. The three news magazines used in this case are Sports Illustrated, MacLean's and the Canadian Geographic. The survey shows that:

a) The probability of selecting at random a person who reads all three magazines is 1/10.
b) 80 do not read any of the three magazines
c) 500 read Sports Illustrated
d) 1/4 read Canadian Geographic
e) 220 read Sports Illustrated and MacLean's
f) The probability of selecting a person who reads Sports Illustrated and Canadian Geographic is 1/6
g) The probability of selecting a person who reads MacLean's given that he or she reads Canadian Geographic is 60%.

Ω (1200)

Sports Illustrated (500)
(S)

MacLean's
(M)
(800)

@= 200   d= 100   f= 520

@= 120

b= 80   c= 60

g= 40

h= 80

(C) (300)
Canadian Geographic

S: "someone who reads Sports Illustrated"
M: "someone who reads MacLean's"
C: "someone who reads Canadian Geographic"

Complete the Venn diagram above and determine the probability of finding someone who reads Canadian Geographic given that he/she reads MacLean's. (10 marks)

ANSWER

a) $P(S \cap M \cap C) = \frac{1}{10}$ of 1200
$\rightarrow \# S \cap M \cap C = 120 \rightarrow @ = 120$

b) $\#(S \cup M \cup C)' = 80 \rightarrow h = 80$

c) $\# S = 500$

d) $\# C = \frac{1}{4}$ of 1200 = 300

e) $\# S \cap M = 220 \rightarrow d = 220 - 120$
$\rightarrow d = 100$

f) $P(S \cap C) = \frac{1}{6} \rightarrow \# S \cap C = \frac{1}{6}$ of 1200
$\rightarrow \# S \cap C = 200 \rightarrow b = 200 - 120$
$\rightarrow b = 80$

Your answer:
$\rightarrow @ = 500 - (100 + 120 + 80)$
$\rightarrow @ = 200$

g) $P(M|C) = 60\%$ or 0.6
$\rightarrow \# M \cap C = 60\%$ of 300
$\# M \cap C = 180$
$\rightarrow C = 180 - 120 \rightarrow c = 60$
$\rightarrow g = 300 - (80 + 120 + 60)$
$\rightarrow g = 40$
$\rightarrow f = 1200 - (80 + 40 + 60 + 80 + 120 + 200 + 100)$
$\rightarrow f = 520$
$\rightarrow \# M = 800$

Therefore, $P(C|M) = \frac{120 + 60}{800}$

$\rightarrow P(C|M) = \frac{180}{800}$ or $\frac{18}{80}$ or $\frac{9}{40}$

or 22.5%

14. The human resources director of a large company sorts *1000* employee files according to age and department.

a) Complete the table below by taking into account the following:
- The probability of selecting an employee under the age of 30 is 50%.
- The probability of selecting an admin employee who is under the age of 30 is 0.02
- The probability of selecting an employee between the age of 30 and 40 given that he/she is an admin employee is 0.5
- The probability of selecting an admin employee given that he/she is above 40 is $\frac{1}{20}$

		Administration (A)	Production (B)	Total
Age < 30	(C)	ⓑ = 20	ⓔ 480	ⓗ = 500
30 ≤ Age ≤ 40	(D)	ⓒ = 30	ⓕ 270	ⓘ 300
Age > 40	(E)	ⓓ = 10	ⓖ 190	ⓙ 200
Total		60	ⓐ = 940	1000

Calculate the following probabilities: (10 marks)

a) Probability of selecting an employee in the production department given that he/she is between the age of 30 and 40 $\longrightarrow P(P|D) = \boxed{270/300 \text{ or } 27/30 \text{ or } 9/10 \text{ or } 90\%}$

b) $P(B|E') = \frac{480+270}{800} = \boxed{\frac{750}{800} \text{ or } \frac{75}{80} \text{ or } \frac{15}{16} \text{ or } 93.75\%}$

c) $P(A|C\cup D) = \frac{20+30}{800} = \boxed{\frac{50}{800} \text{ or } \frac{5}{80} \text{ or } \frac{1}{16} \text{ or } 0.0625 \text{ or } 6.25\%}$

ANSWER

ⓐ = 1000 − 60
→ⓐ = 940

① $P(C) = 50\%$
→ #C = 50% of 1000
→ #C = 500
→ ⓗ = 500

② $P(A\cap C) = 0.02$
→ #A∩C = 0.02 of 1000
→ #A∩C = 20
→ ⓑ = 20

③ $P(D|A) = 0.5$
→ #A∩D = 0.5 of 60 = 30
→ ⓒ = 30

→ⓓ = 60 − (20+30)
→ⓓ = 10

④ $P(A|E) = \frac{1}{20} = \frac{10}{ⓙ}?$

→ ⓙ = 200
→ ⓘ = 1000 − (200+500)
→ ⓘ = 300
→ ⓔ = 500 − 20
→ ⓔ = 480
→ ⓕ = 300 − 30
→ ⓕ = 270
→ ⓖ = 940 − (480+270)
→ ⓖ = 190